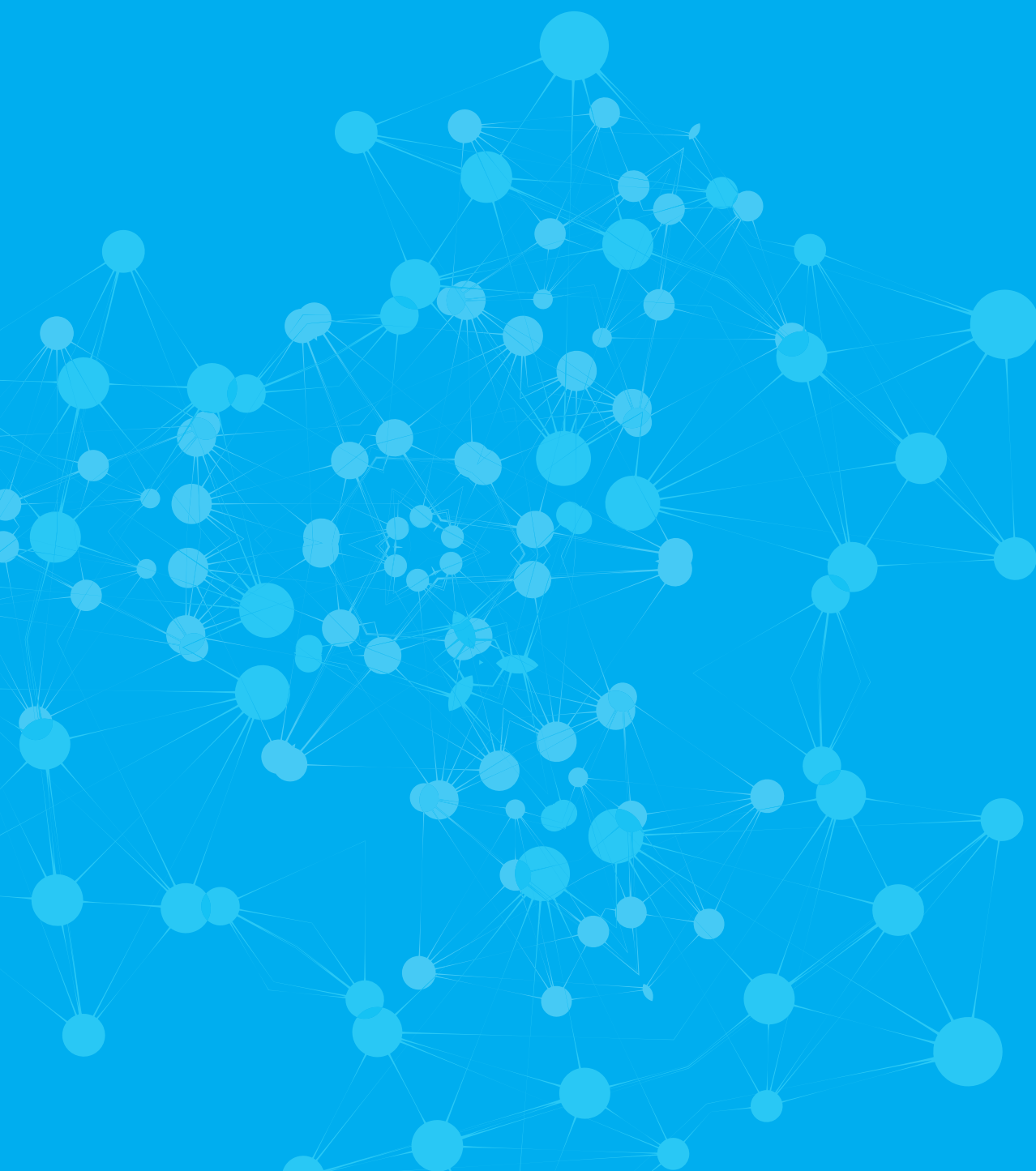


NUMERACY COMPETENCE WORKSHOPS

An Innovative Toolkit for the Development of the Digital & Numeracy Competence for Low Skilled Adults (IntoDIGITS)

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Active Citizenship Level 1

LEVEL 1

Workshop Title: Value of basic numbers

CODE: L1.WA1.N1

LEVEL 1

WORK AREA 1: NUMBERS

PARTNER: FEH

Duration: 5 hours

Learning Outcomes Covered

Work Area 1: NUMBERS

Unit 1.1 Integers & Whole Numbers

LO1: Read and write Arabic numerals: 0 1 2 3 4 5 6 7 8 9

LO2: Token entry and word record of a two-digit number

LO3: Token entry and word record of a three-digit number

LO4: Token entry and word record of a four-digit number

LO5: Read and write numbers with use of symbols "I" and "V"

LO6: Read and write numbers with use of symbols "I", "V", and "X"

LO7: Read and write numbers with use of symbols "I", "V", "X", and "C"

LO8: Read and write numbers with use of symbols "I", "V", "X", "C", and "L"

Unit 1.2 Basic Calculations with whole numbers

LO1: Addition of natural numbers within the range of 20 (so-called endings).

LO2: Subtraction of natural numbers within the range of 20.

LO3: Addition and subtraction within the scope of 100 full tens.

LO4: Subtraction of natural numbers within the range of 20.

LO5: Addition and subtraction of one-digit numbers to two-digit numbers with crossing of the decimal threshold.

LO6: Long addition and subtraction of any numbers.



LO7: Interpretation of multiplication as the addition of equal components

LO8: Multiplication of numbers within the range of 100 (multiplication table)

LO9: Division of numbers within the range of 100

LO10: Divisibility of natural numbers (by 2, 5, 10)

Unit 1.8 Ordering Whole numbers

LO1: Being able to compare two numbers (Greater/smaller than, equal: $<$ $>$ $=$)

LO2: Classify numbers

LO3: Recognize equal numbers

LO4: Able to sort numbers

LO5: Arrange natural numbers in increasing or decreasing order

LO6: Count forward from a given number

LO7: Count forward or backward

LO8: Skip counts (e.g. by twos)

LO9: Count from a given number

Short introduction to the workshop

This workshop includes 4 activities. The first activity is a **Mixed Bingo** that aims at providing an understanding and identification of Roman and Arabic numbers. The second is a low level **Sudoku** where students will have to play a normal Sudoku as usual. The former is aimed at teaching students the order of numbers and how they can interact. However, this is a collaborative Sudoku where each team/student's opinion is taken into account. The activity "**Value of words**" helps calculating prices and basic numbers in a context where they can use in a daily situation. Finally, the **crossword** activity, deals with basic numeracy and how to spell it or write it.

How the workshop (its activities) teach the LOs to the participants

This workshop aims to provide students with a clear understanding of roman and Arabic numbers and their differences, basic calculations like addition, subtraction and comparison, multiplication and division among basic figures and numbers, spelling and daily calculations.

- **Mixed Bingo.** For a student to understand what it is going on in a friendly Bingo, they need to know in depth what each letter in the Roman alphabet means. With a play such as this, students calculate their meaning in order to win. By participating, basic calculations and convert Arabic and roman



numbers to one another without even noticing while having fun. If necessary, a reminder of the basic rules will be necessary.

- **Collaborative Sudoku.** The basic order of numbers can be understood. It also improves the student's memory with numbers and how to deal with them in an easy and comprehensive way with a clear set of rules. This should be reminded as well in case students and teachers do not remember how to play it. It also improves quick-thinking, concentration and number memory. The extra added, the collaboration among students, help them discuss and see whether the numeric rules have been followed.
- **Value of words.** Helps developing basic math calculation skills such as subtraction and addition. Moreover, the activity does it in a daily scenario that may be applied to real situations.
- **Number crosswords.** Some adults might recognize numbers but not be able to apply the written rules of them. When reading papers or other kind of contracts they might lack the skills to identify them.

Activities below help students to get to know the different types of numbers, put them under situations where basic math calculations must be done in order to succeed and where they can place them correctly.

Challenges during the implementation of activities and how can be addressed

Lack of understanding of some of the games/plays and the rules that are part of the learning process. Reluctant to participate in the game out of shyness or a degree of difficulty unmotivating. If required, trainers should check online the extra resources included.



ACTIVITY 1: Mixed Bingo

This activity aims to provide a thorough understanding of Roman and Arabic Numbers. In practice, it is similar to a Bingo Game. However, instead of just Arabic Numbers, both Arabic and Roman Numbers are used. Make sure every student has a Bingo Card (**Annex 1**) and if necessary, provide a list of the Roman Numbers and its meaning in Arabic numbers. A pair of students may be the speakers. They will say out loud the numbers picked up from the box/bag. You might include rewards as a prize for the winners.

Step1: Provide each student with a table filled with numbers which has been already pre-established in the annexes (**Annex 1**). Depending on the number of students, they can be set in pairs.

Step 2: Choose a student or a pair of students. They will be the speakers. Ask them to step up in front of the class in order to say out loud the numbers picked up..

Step 3: Ask (the speakers) to take one number of the bag of folded paper in the box/bag. They need to say out loud the number. Each number might be either in Roman or Arabic style. Every time a number is mentioned, the speakers must cross the numbers in the big square (**Annex 2**).

Step 4: Each pair or student has to cross each of the numbers said out loud. If a student(s) get all done they win.

Step 5: Bingo can be repeated as many times as possible.

Resources for Trainer

- Annexes 1, 2 and 3.

Other Resources needed

- Annex 1, 2 and 3 (1 and 2 for each participant, 3 for the whole group)
- A Training Room
- Bag or box to be filled out with the folded papers with folded papers to be picked up. This should contain mixed numbers (Arabic and Roman).

ACTIVITY 2: Collaborative Sudoku's

This activity aims to provide a practical knowledge applicable to basic numbers, arrange natural numbers in increasing or decreasing numbers and the ability of sorting numbers. Students set in pairs have to fill out Sudoku's squares ([Annex 3](#)). Each student in each pair fills out the squares, following Sudoku's rules. Both students in a pair must agree on each move.

Step1: Provide each student with a Sudoku table(s) with some of its squares filled ([Annex 3](#)). Solutions for the trainer have been included. Set students in pairs.

Step 2: Students need to fill out the boxes by following the rules of the traditional Sudoku. Trainer may hand out the written rules to each pair of students to make things easier. Pairs of students may compete with one another. They can compete either by filling as many gaps as possible or complete them all.

Step 3: Trainer must set a limited number of moves for each pair of students. The number of moves must be at least the same than the blanks to be filled. Each student has one move at a time.

Step 4: Trainer must check whether the moves are in line with the rules of Sudoku. Each move of each student has to be approved by the other student in the pair. Both students must agree with the final version of the Sudoku. If they do not, they cannot finish the game.

Step 5: Once finished, each pair must switch with another pair of students. Each pair of student rates and comments in front of everyone else the Sudoku assigned.

Step 6: Trainer must assess which has been the winner.

Resources for Trainer

- Annexes 3 and 4.
- More Sudokus and different kind of levels can be found here:
<https://www.sudoku-online.org/imprimir-sudokus.php>
- Basic rules of Sudokus <https://sudoku.com/how-to-play/sudoku-rules-for-complete-beginners/>

Other Resources needed

- Annex 3 and 4. Annex 4 can be handed out to students as well.
- A Training Room

ACTIVITY 3: Value of Words

This activity aims to provide a practical exercise for students to get to know basic calculations with whole numbers (addition and subtraction) including decimals.

Step1: Provide each student with a table(s)/boxes filled with the alphabet. Each letter should have a euro value (**Annex 4**). You can change and assign different values if you wish.

Step 2: Give students **Annex 5**. This has several questions. Ask them to answer them. Students may do it individually or in groups and share their answers. You can add more questions if required.

Step 3: Then, each group or student must present results in front of the class.

Step 4: There is a second part of the activity. Split the class into 4 groups.

Step 5: Provide each group with **Annex 6**, the items with their price tags.

Step 6: Use a timer and provide 2 minutes to all groups in order to calculate which words and their value could use to purchase any of the products. They can purchase as many items as possible.

Step 7: The team that purchases more products with more words wins.

Resources for Trainer

- Annexes 4, 5 and 6

Other Resources needed

- Annex 4, 5 and 6. Some of them can be handed out to students as well.
- A Training Room.
- A timer.
- A horn.
- White board and markers.

ACTIVITY 4: Number Crosswords

This activity aims to provide a practical exercise to get to know basic numbers and how to write them. It is a crossword student must fill with the written version of each number. Trainers may create their own crosswords with the resources available.

Step1: Provide each student with **Annex 7**.

Step 2: Ask them to fill out the crossword.

Step 3: You may change the scenarios in each case or check the external resources to create different number crosswords.

Resources for Trainer

- Annex 7
- External resources available to form your own numbers crosswords here <https://www.boatloadpuzzles.com/playcrossword>

Other Resources needed

- Annex 7.
- A Training Room.
- White board and markers.

ANNEXES

Annex 1

		XXIX	XXXIII		LII	LXVI		LXXXIII
			XXXVI	XLIV			LXXIII	LXXXVI
II	XII			XLIX		LXIX	LXXIV	

III		XXIII		XLI		LXI		LXXXI
	XVIII		XXXVIII	XLVII		LXV	LXXII	
VII		XXVII			LV		LXXIX	LXXXVII

		XXI		XLII	LIII		LXXVIII	LXXXV
IV		XVI	XXXII		LVIII			LXXXVIII
V	X		XXXIX	XLVI		LXIII		

	XIII		XXXI	XL		LXVII	LXXI	
	XIV	XX			L		LXXVII	LXXXIX
VIII		XXII		XLV	LIX			XC

VI	XV			XLIII		LXII	LXX	
	XVI		XXXIV		LI			LXXX
		XXIV	XXXVII		LIV	LXVIII	LXXV	LXXXIV

I	XVII		XXX		LVI	LX		
IX		XXV	XXXV			LXIV	LXXVI	
	XIX	XXVIII		XLVIII	LVII			LXXXIII



Annex 2

I	II	III	IV	V	VI	VII	VIII	IX	X
XI	XII	XIII	XIV	XV	XVI	XVII	XVIII	XIX	XX
XXI	XXII	XXIII	XXIV	XXV	XXVI	XXVII	XXVIII	XXIX	XXX
XXXI	XXXII	XXXIII	XXXIV	XXXV	XXXVI	XXXVII	XXXVIII	XXXIX	XL
XLI	XLII	XLIII	XLIV	XLV	XLVI	XLVII	XLVIII	XLIX	L
LI	LII	LIII	LIV	LV	LVI	LVII	LVIII	LIX	LX
LXI	LXII	LXIII	LXIV	LXV	LXVI	LXVII	LXVIII	LXIX	LXX
LXXI	LXXII	LXXIII	LXXIV	LXXV	LXXVI	LXXVII	LXXVIII	LXXIX	LXXX
LXXXI	LXXXII	LXXXIII	LXXXIV	LXXXV	LXXXVI	LXXXVII	LXXXVIII	LXXXIX	XC
XCI	XCII	XCIII	XCIV	XCV	XCVI	XCVII	XCVIII	XCIX	C



Annex 3

	8		5	7	6	2		
			4		2			
				3	9	5	4	8
6	3		9			8	5	2
	9		2			3	7	
8				5		6	9	4
2	5	7	6		3	4	8	9
3		8	7				2	5
	4							6

9	8	4	5	7	6	2	1	3
5	1	3	4	8	2	9	6	7
7	2	6	1	3	9	5	4	8
6	3	1	9	4	7	8	5	2
4	9	5	2	6	8	3	7	1
8	7	2	3	5	1	6	9	4
2	5	7	6	1	3	4	8	9
3	6	8	7	9	4	1	2	5
1	4	9	8	2	5	7	3	6

	5	8		6	4			
6		9					4	
		2	3	7			6	8
	8	7					5	
		3	8			7		
5			4		7			2
8	7			4	6			
			2	5	3	8		
	2			8				4

7	5	8	1	6	4	9	2	3
6	3	9	5	2	8	1	4	7
4	1	2	3	7	9	5	6	8
1	8	7	6	3	2	4	5	9
2	4	3	8	9	5	7	1	6
5	9	6	4	1	7	3	8	2
8	7	1	9	4	6	2	3	5
9	6	4	2	5	3	8	7	1
3	2	5	7	8	1	6	9	4

		4	6		8	5		
					7	6	2	9
6				2	5	8	4	7
	9	3	7		2		6	5
	7	5					8	
2	6	8			4			3
		6	8		9	7		2
	8			3		4		6
		2		7		3	5	8

7	2	4	6	9	8	5	3	1
8	5	1	3	4	7	6	2	9
6	3	9	1	2	5	8	4	7
4	9	3	7	8	2	1	6	5
1	7	5	9	6	3	2	8	4
2	6	8	5	1	4	9	7	3
3	4	6	8	5	9	7	1	2
5	8	7	2	3	1	4	9	6
9	1	2	4	7	6	3	5	8

		2			8			
4	3		7		2	8		
	6		5	9	4	3		
		7		2	3			6
3	2	5	6	4		9	8	7
6			9			1	2	3
	1	6	4	3		7	9	8
				7			3	
	4	3	2	8	9		1	5

5	7	2	3	1	8	4	6	9
4	3	9	7	6	2	8	5	1
8	6	1	5	9	4	3	7	2
1	9	7	8	2	3	5	4	6
3	2	5	6	4	1	9	8	7
6	8	4	9	5	7	1	2	3
2	1	6	4	3	5	7	9	8
9	5	8	1	7	6	2	3	4
7	4	3	2	8	9	6	1	5

		3	6			4	8	7
	7			3	4			5
4	5			7	8			
	3			6	7	9	2	8
6				4		5	3	
			5			7	4	
9	4	5		8				2
	8			5	2	3	6	9
3		2				8	5	4

2	9	3	6	1	5	4	8	7
1	7	8	2	3	4	6	9	5
4	5	6	9	7	8	2	1	3
5	3	4	1	6	7	9	2	8
6	2	7	8	4	9	5	3	1
8	1	9	5	2	3	7	4	6
9	4	5	3	8	6	1	7	2
7	8	1	4	5	2	3	6	9
3	6	2	7	9	1	8	5	4

	5			8	7			6
		4						2
		9	3	2		7	5	4
9		5						7
	2			6				
6	4	7	2					
7		3	8			6		5
			9	7		3	4	8
4		2	5					

2	5	1	4	8	7	9	3	6
3	7	4	6	5	9	8	1	2
8	6	9	3	2	1	7	5	4
9	3	5	1	4	8	2	6	7
1	2	8	7	6	5	4	9	3
6	4	7	2	9	3	5	8	1
7	9	3	8	1	4	6	2	5
5	1	6	9	7	2	3	4	8
4	8	2	5	3	6	1	7	9



Annex 4

A	B	C	D	E	F	G
H	I	J	K	L	M	N
O	P	Q	R	S	T	U
V	W	X	Y	Z		

1€	2€	3,5€	4,8€	5€	6€	7,5€
8€	9€	10,3€	11€	12€	13€	14,5€
20€	22€	23€	30,5€	35€	40€	44,3€
50€	52,5€	60€	70€	80,8€	90,2€	99,7€



Annex 5

1. How much is your first name worth?
2. How much is your family name worth?
3. Who has the most expensive name in your family?
4. Can you find any words that are worth exactly 100€?
5. Can you find some very expensive 2 letter words? And 3 letter words?
6. Can you find some very cheap 2 letter words? And 3 letter words?
7. Who has the cheapest name among you?

You can add further questions if needed.

Annex 6



(20€)



(15€)



(17,5€)



(250€)



(1.200€)



(348€)





(359€)



(189€)



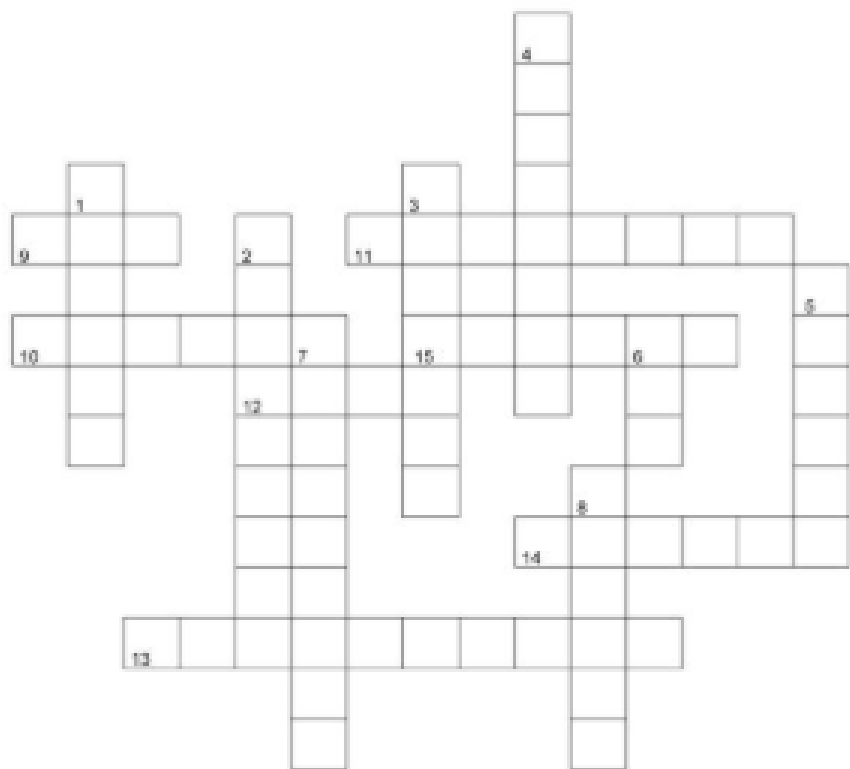
(150€)



(1.250€)



Annex 7



Down:	Across:
1. 12	9. 2
2. 26	10. 11
3. 15	11. 56
4. 14	12. 9
5. 40	13. 67
6. 10	14. 80
7. 92	15. 20
8. 90	



LEVEL 1

Workshop Title: Dealing with negative numbers, shares and quantities.

Code: L1.WA1.N2

LEVEL 1

WORK AREA 1: NUMBERS

Partner: FEH

Duration: 5 hours

Learning Outcomes Covered

WORK AREA 1: NUMBERS

Unit 1.5. Negative numbers and basic calculations.

LO1: Recognise concept of the negative number. Opposite numbers (thermometer, number line).

Unit 1.6 Working with Ratio & Proportions.

LO1: Understanding basic concept of ratios and proportions

LO2: Being able to describe shares/quantities

LO3: Recognise which share is bigger/smaller

LO4: Divide a whole figure in different shares

Short introduction to the workshop

This workshop includes 4 activities. A brief description of each of them can be found here.

Negative Number War: objective is for students to have clearer the differences between negative and positive numbers. **Compare and Draw:** with the help of the value of the x in the proportions given, students have to draw and paint a given drawing. **Shopping with Proportions:** an activity that aims at helping understand basic proportions in daily situations such as shopping. Real life situations are provided for students to calculate the best value of products. **Unit rates and proportions when shopping:** students need to guess what the better buy is when going shopping.

How the workshop (its activities) teach the LOs to the participants

This workshop aims to provide students with a clear understanding on negative numbers, basic calculations and the relation of shares and quantities. There are numerous ways to teach Shares:

- **Negative Number War.** By adding and subtracting the numbers on the dice and placing them on the line paper, students may get a perspective of what negative and

positive numbers mean and how they are related. It is a basic adding of negative numbers that may help students with low level of knowledge on the issue the relation among positive and negative numbers and how to add them.

- **Compare and Draw.** By drawing and painting students may get to understand the relation between numbers and comparison between fractions.
- **Shopping with proportions:** by comparing responses and prices of several products, students realize of the use of fractions and units and how to use it for real-life situations.
- **Daily situations when shopping:** This activity aims at providing real life situations for students to know what is required to shop for the best value. Several Annexes are used under this activity.

After this, a quiz can be found. The activities are focused on basic comparison among ratios so to complete the whole package of exercises.

Challenges during the implementation of activities and how can be addressed

Unwillingness of students to participate in the workshops or games. Too complex calculations depending on the differences of level among students participating.

It is important for the trainer to remind several basic concepts if necessary. Have a look at the other sources for the trainer section for further details.



ACTIVITY 1: Negative Number War

This activity aims to provide practical exercise so to understand differences between negative and positive numbers and how to scale them. Teachers should draw a number line on paper/board. Place a counter in number 14. Rules of the exercise: there are two players. One is Plus and one is called Minus. *Plus* moves the counter from left to right and *Minus* moves the counter from right to left. Every time a student rolls the dice, both numbers must be added up. Use the result to move the counter on the line drawn.

Step1: Draw a number line on paper like shown on **Annex 1**, and place the counter on number 14.

Step 2: Ask students to roll the dice. Student should throw the two dice and add up the two numbers. Ask students to add or substrate the two numbers.

Step 3: Ask students to move the counter the same number of times than the results in step 2. If the counter has reaches -10, Minus has won and so, of course if the counter reaches 27, student labeled Plus has won.

Step 4: There might be some variations of the game. You may have one counter for each student and see who reaches their end first. You may use three dice. You may use one dice and a shorter line.

Variations enrich the game and may add further learning outcomes.

Other variation: this time, when you throw the dice, you can decide whether to add, subtract, multiply or divide the numbers on the dice. You must reach -13 or 13 exactly to win. You can introduce further variations.

Resources for Trainer

- Annex 2.
- Video: <https://www.khanacademy.org/math/arithmetic/arith-review-negative-numbers/arith-review-neg-num-intro/v/negative-numbers-introduction>

Other Resources needed

- Annex 2
- Two dice
- Counters
- A Training Room

ACTIVITY 2: Compare and Draw

This activity wants to help understand proportions and comparison between fractions. Students must solve the equations and draw according to the Annexes.

Step 1: Give each student or pair of students **Annex 2**.

Step 2: Each pair of students must decide which number goes in the “x” place. You may need to give lessons about fractions, comparisons and proportions. You can find further details in *resources for trainer*.

Step 3: Once they solve the equations, ask them to paint the drawing depending on the colors assigned. Check out **Annex 3**

Step 4: You can change the rules as you wish. Templates are available as Annexes. Solutions for Annex 3 are found in **Annex 4**.

Resources for Trainer

- Annex 3, Annex 4 and Annex 5.
- **Video:** <https://es.khanacademy.org/math/pre-algebra/pre-algebra-ratios-rates/pre-algebra-write-and-solve-proportions/v/find-an-unknown-in-a-proportion>
- **Video about Fractions:** <https://www.khanacademy.org/math/arithmetic/fraction-arithmetic>

Other Resources needed

- A Training Room.
- Pens

ACTIVITY 3: Proportions and rates at the grocery store

This activity aims at providing real life situations where students may have real world knowledge of what is required for the best value. Students may use basic calculations with fractions to solve the problems ahead.

Step 1: Provide to each of the students or pair of students with **Annex 5**.

Step 2: Ask student to form pairs of student. Once they have solved which one in each case is a better deal. Ask them to compare responses.

Step 3: Each pair of students should explain in front of the whole class the options chosen. If there are disagreements, encourage students to explain why.

Step 4: Give to each pairs of students **Annex 6**. Ask them to answer the questions attached. In **Annex 7** you can check the answers.

Step 5: Go back to step 3. Each pair of students should explain in front of the whole class the options chosen. If there are disagreements, encourage students to explain why.

Resources for Trainer

- Annex 5, 6 and 7.
- **Ratios, rates and proportions video:** <https://es.khanacademy.org/math/pre-algebra/pre-algebra-ratios-rates/pre-algebra-write-and-solve-proportions/v/find-an-unknown-in-a-proportion>
- **Ratios, rates and proportions video about Fractions:** <https://www.khanacademy.org/math/arithmetic/fraction-arithmetic>

Other Resources needed

- A Training Room.
- Computer Internet Access
- Poster paper
- Markers
- Calculators

ACTIVITY 4: Daily situations when shopping!

This activity aims at providing real life situations for students to know what is required to shop for the best value. Several Annexes are used under this activity.

Step 1: Give each student **Annex 8**. Ask students to choose a product that has different sizes.

Step 2: For the next part of the activity, provide students with **Annex 9**. Using the products before, ask students to calculate the cost if they had bought each item once a week for an entire year. Then, students should calculate how much they would save per year by buying the cheaper one. Provide guidance if necessary.

Step 3: Ask students to tell to the rest of the class their calculations.

Step 4: Give **Annex 10**. These are a series of questions concerning rate unit, proportions and its relations. Students need to ask correctly in groups or pairs.

Step 5: Ask all pairs of students to discuss each decision. They need to provide which basic calculations they have made.

It is likely that you will need to remind basic concepts if needed.

Resources for Trainer

- Annex 8, 9 and 10.
- Video: Ratios, rates and proportions <https://www.khanacademy.org/test-prep/sat/sat-math-practice/new-sat-problem-solving-data-analysis/v/sat-math-q1-easier>

Other Resources needed

- A Training Room.
- Poster paper
- Markers
- Calculators



ANNEXES

Annex 1

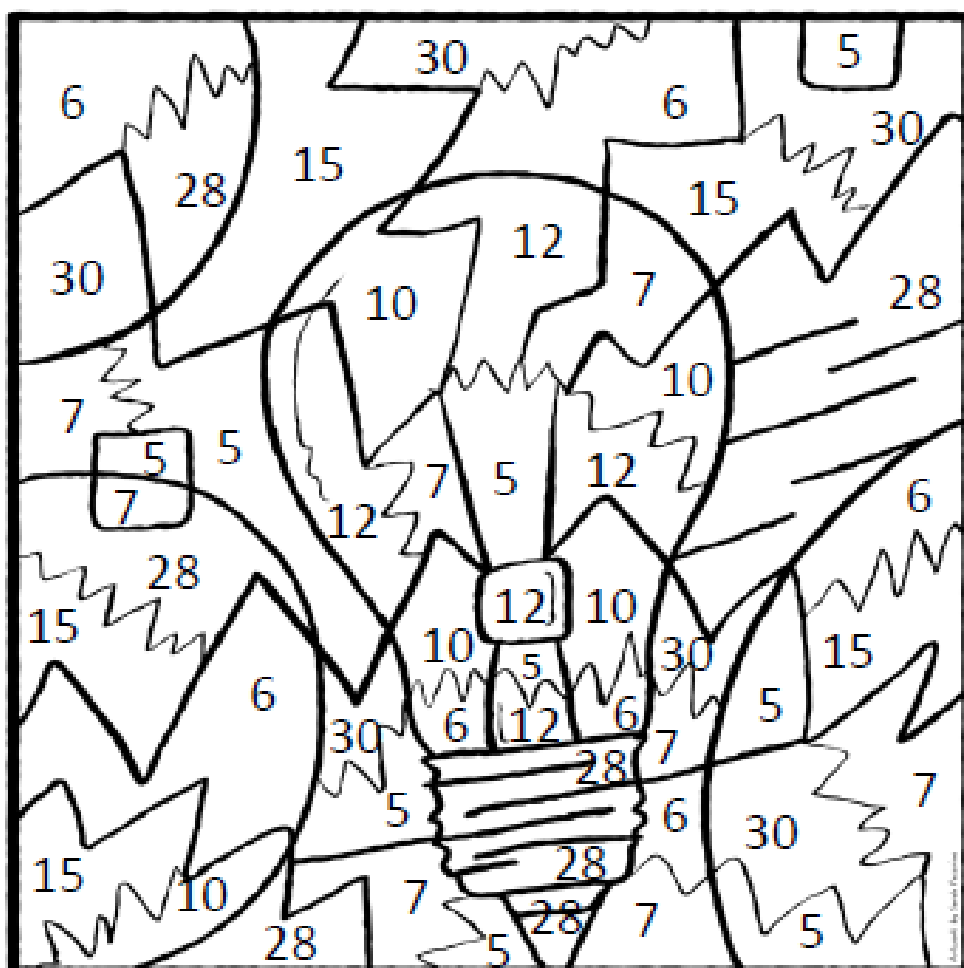


Annex 2

Solve for x	x=	Color it...
$\frac{2}{3} = \frac{4}{x}$		blue
$\frac{10}{12} = \frac{25}{x}$		green
$\frac{14}{20} = \frac{x}{40}$		purple
$\frac{2}{5} = \frac{x}{25}$		yellow
$\frac{8}{x} = \frac{2}{3}$		orange
$\frac{x}{45} = \frac{3}{9}$		red
$\frac{11}{44} = \frac{x}{20}$		yellow
$\frac{4}{x} = \frac{12}{21}$		gray



Annex 3



Annex 4

Solve for x	x=	Color it...
$\frac{2}{3} = \frac{4}{x}$	x=6	blue
$\frac{10}{12} = \frac{25}{x}$	x=30	green
$\frac{14}{20} = \frac{x}{40}$	x=28	purple
$\frac{2}{5} = \frac{x}{25}$	x=10	yellow
$\frac{8}{x} = \frac{2}{3}$	x=12	orange
$\frac{x}{45} = \frac{3}{9}$	x=15	red
$\frac{11}{44} = \frac{x}{20}$	x=5	yellow
$\frac{4}{x} = \frac{12}{21}$	x=7	gray



Annex 5

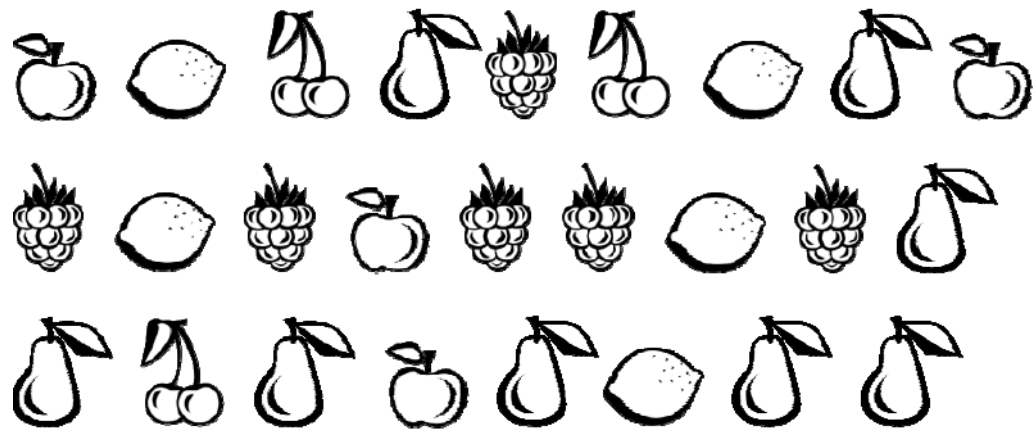
You go shopping and find all the items below. In each case, circle which set of items is a better deal (has the best unit price) and discuss it with your partner. After that, share results with the rest of the class.

12 bags of pork rinds for 2€	Or	8 sticks of gum for 0,8€
4 DVDs for 12.5€	Or	3 Blue Rays for 8,5€
2 headphones for 300€	Or	1 speaker for 450€
15 songs for 11.5€	Or	20 CDs for 14.5€
11 apples for 4,25€	Or	6 pears for 2€
3 tires for 180€	Or	5 tires for 350€
15 packs of cards for 32€	Or	30 packs of cards for 65€
3 car whases for 25€	Or	5 car whases for 40€
6 fotball tickets for 45€	Or	9 hocket tickers for 123€
22 rolls of towels for 10€	Or	40 rolls of towels for 15€
24 pens for 11€	Or	50 pens for 20€



Annex 6

At the end of the day, Mrs. Davis’ local farmers market stan had all the left over fruit that you may see below. At the begining of the day, Mrs David had 50 pieces of each type of fruit (apples, lemons, cherries, pears and grapes).



Which is the least popular (in sales) fruit for the day?

What is the ratio of left over cherries to grapes?

What is the ratio of leftover (in simplest form):

- A) Apples to pears
- B) Lemons to grapes

If the same exact amount of fruit was sold every day, how many pieces of each type of fruit should Mrs David have in stock daily?

Apples:

Lemons:

Pears:

Cherries:

Grapes:



Annex 7

Which is the least popular (in sales) fruit for the day? Pears (42 sold)
What is the ratio of left over cherries to grapes? 3 to 6 or 1 to 2
What is the ratio of leftover (in simplest form): A) Apples to pears: 4 to 8 or 1 to 2 B) Lemons to grapes: 5 to 6
If the same exact amount of fruit was sold every day, how many pieces of each type of fruit should Mrs David have in stock daily? Apples: 4 Lemons: 5 Pears: 3 Cherries: 6 Grapes: 8 Amount needed would be 50 minutes amount leftover, apples (46), lemons (45), pears (42), cherries (47), grapes (44).



Annex 8

Item	Product 1	Product 2	Which is the better buy?
Example: Cheese Ham	Price: 5€/8 slice Unit Rate: 0,625€	Price: 5€/7 slice Unit Rate: 0,74€	Ham, because it is 10 cents cheaper
	Price: Unit:	Price: Unit:	
	Price: Unit:	Price: Unit:	
	Price: Unit:	Price: Unit:	
	Price: Unit:	Price: Unit:	
	Price: Unit:	Price: Unit:	
	Price: Unit:	Price: Unit:	



Annex 9

Item	Size 1	Size 2	How much would you save?
	Price Price per year:	Price Price per year	
	Price Price per year	Price Price per year	
	Price Price per year	Price Price per year	
	Price Price per year	Price Price per year	
	Price Price per year	Price Price per year	
	Price Price per year	Price Price per year	



Annex 10

Jeanie sells caramels out of her giant bag at 4 for 5 cents. The machine at the store sells you 9 for 25 cents. Which is the better deal for you? Show your basis calculations.

Tom sells cards at 10 for 35 cents. Is that a better deal than 12 for 40 cents? Prove your thinking

The hardware store sells sparklers for the Carnival. They charge 19 cents apiece. The fireworks stand charges 85 for four. Which is the better deal? How can you tell?

A well known cereals brand has 11 grams of sugar in each 1 ounce serving. Another one has 13 grams of sugar in each 1.4 ounce serving. Which one has less sugar for an ounce of cereal? How can you tell? Prove your answer.

Active Citizenship Level 2

LEVEL 2

WORK AREA 1: NUMBERS

Workshop Title: Basic calculations with Negative Numbers

CODE: L2.WA1.N1

LEVEL 2

WORK AREA 1: NUMBERS

Partner: FEH

Duration: 5 hours

Learning Outcomes Covered:

WORK AREA: NUMBERS

Unit 1.5: Negative numbers and basic calculations.

LO1: Add negative numbers.

LO2: Subtract negative numbers.

LO3: Make multiplication and divide negative numbers.

LO4: Operations on negative numbers.

Short introduction to the workshop

This workshop includes 2 activities. A brief description of each of them can be found here. **Connect Three Numbers:** this activity aims to provide practical game so to understand how to calculate negative numbers specifically add and subtract both types of numbers. Students may practice basic calculations with negative and positive numbers. **Find out the hidden message:** basic calculations with negative numbers such as multiplication, division, adding and subtracting in a board with hidden messages that student have to find out.

How the workshop (its activities) teach the LOs to the participants

This workshop aims to provide students with a clear understanding on negative numbers, basic calculations and the relation of shares and quantities. There are numerous ways to teach Shares:

- **Connect tree numbers.** They need to make calculations so to line three numbers in the board given. This makes student think about probabilities of calculation with negative and positive numbers. Students can reflect about which calculations were easier, which numbers they wanted to pursue and which numbers are most difficult to get. Students aim is to get at least three in a row in the board showed in Annex 1.
- **Find out the hidden message.** It has the main objective of understand the basic calculations (subtracting, adding, multiplication and division) with negative numbers. It mixes several types of calculations. Messages can be modified depending on the



wish of the trainer as well. This boosts competition and appeals for learning in order to beat the other team of students. By doing this exercise, students understand how simple calculations are made.

After this, a quiz can be found. These are about basic comparisons between ratios.

Challenges during the implementation of activities and how can be addressed

Unwillingness of students to participate in the workshops or games. Too complex calculations depending on the differences of level among students participating.



ACTIVITY 1: Connect Three Numbers

This activity aims to provide practical game so to understand how to calculate negative numbers, specifically, addition and subtraction in both cases of numbers. The activity is played with two dices, one will represent the numbers 1,2,3,-4,-5,-6 and the other dice the numbers -1,-2,-3,4,5,6. The main objective is for students to place three of the counters in a straight line in **Annex 1** before the rival does.

Step1: Give students **Annex 1**.

Step 2: Each student/player rolls the dice. Students decide if they want to add or subtract both numbers. They need to get any of the numbers of the board in **Annex 1**.

Step 3: Students cannot cover a number which has already been covered. If they are unable to find a total which has not been covered they must pass.

Step 4: Students may play the game a few times. After the game is finished. Ask several questions. **1.** Are there some numbers that we should be aiming for? Why? **2.** Which number on the grid is the easiest to get? Why? **3.** Which number is the most difficult to get? Why?

Resources for Trainer

- Annexes 1 and 2.
- **Video:** <https://es.khanacademy.org/math/cc-sixth-grade-math/cc-6th-negative-number-topic>

Other Resources needed

- Annex 1 and 2 (Playing Board and Questions)
- Two dice
- Counters, two colours.
- A Training Room

ACTIVITY 2: Find out the hidden message!

This activity has the main objective of understand the basic calculations (subtracting, adding, multiplication and division) in a funny way. Students or pairs of students have to solve all calculations. Student then have to match the result with the correspondent alphabet word. Students that find out the message, win.

Step 1: Provide to each of the students or pair of students with **Annex 2**.

Step 2: Set 2 minutes to solve equations. Check them with students. The first in solving the hidden message, wins.

Step 3: You might find several similar exercises in **Annex 3** and **Annex 4**. Solutions of both activities can be found on **Annex 5**.

Step 4: You may, after the exercise, compare among students results. Ask them which one has been the most difficult and why.

Resources for Trainer

- Annex 6, Annex 7, Annex 8, annex 9
- **Adding and Subtracting video:**
<https://www.khanacademy.org/math/arithmetic/arith-review-negative-numbers/arith-review-sub-neg-intro/v/adding-and-subtracting-negative-number-examples>
- **Multiplication and Division video:**
<https://www.khanacademy.org/math/arithmetic-home/negative-numbers/mult-divide-negatives/v/multiplying-and-dividing-negative-numbers>

Other Resources needed

- A Training Room.
- Pens

ANNEXES

Annex 1

-12	-11	-10	-9	-8
-7	-6	-5	-4	-3
-2	-1	0	1	2
3	4	5	6	7
8	9	10	11	12



Annex 2

A	B	C	D	E	F	G	H	I	J	K	L	M
-6	-7	1	-1	-12	3	11	15	-4	13	0	9	-10
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
10	2	-5	6	7	-15	-2	-9	-14	8	-3	-11	-18

$3 - 7$	$-10 + 8$	$-3 + 11$	$-2 - 4$	$-7 - 8$	$12 \div -2$

$(-6) - (-7)$	$(-5) + (-7)$	$(+5) - (-2)$	-4×3	$(+2) - (+8)$	$(+4) - (-5)$

$2 - 7 + 3 + 2$	$-5 - 2 + 3$	-3×-3	$(+3) - (-7) + (-1)$	$(-3) + (-7) + (-2)$	$(-2) - (-3) - (-6)$



Annex 3

A	B	C	D	E	F	G	H	I	J	K	L	M
-6	-7	1	-1	-12	3	11	15	-4	13	0	9	-10

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
10	2	-5	6	7	-15	-2	-9	-14	8	-3	-11	-18

3-7	-10 + 8	-3+11	-2-4	-7-8	4-10
$(-6) - (-7)$	$(-5) + (-7)$	$(+5) - (-2)$	$-(-15) - (-3)$	$(+2) - (+8)$	$(+4) - (-5)$
2-7+3+2	-5-2+3	-2+7+4	$(+3) - (-7) + (-1)$	$(-3) + (-7) + (-2)$	$(-2) - (-3) - (-6)$



Annex 4

A	B	C	D	E	F	G	H	I	J	K	L	M
-6	-7	1	-1	-12	3	11	15	-4	13	0	9	-10

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
10	2	-5	6	7	-15	-2	-9	-14	8	-3	-11	-18

-3×-5	-3×-4	$--3 \times 5$	$--18 / 2$	$--8 / -8$	$(-4)^2$
$36 / -3$	$(-1)^2$	$-4 - 6 / 5$	$5 \times (-6) / -2$	$2 + 7 \times (-2)$	$(-3)^2$
$-3 \times -8 / -6$	$2 \times -3 + (-3)^2$	$-4(-2 + 5)$	$3 - 11 / -4$	$(-6)^2 / -4$	$20 - (-2 \times -11)$
$-2 + 3 \times (-4) / 7$	$-7 - 8 / -2 - 3$	$(-4)^3 / (-4)^2$	$-10 + -16 + -2$		



Annex 5

3-7	-10 + 8	-3+11	-2-4	-7-8	4-10
I	T	W	A	S	A
$(-6) - (-7)$	$(-5) + (-7)$	$(+5) - (-2)$	$-(-15) - (-3)$	$(+2) - (+8)$	$(+4) - (-5)$
C	E	R	E	A	L
2-7+3+2	-5-2+3	-2+7+4	$(+3) - (-7) + (-1)$	$(-3) + (-7) + (-2)$	$(-2) - (-3) - (-6)$
K	I	L	L	E	R

-3 x -5	-3 x -4	--3 x 5	--18 / 2	--8 / -8	$(-4)^2$
H	E	S	U	C	K
36/-3	$(-1)^2$	-4-6/5	$5x(-6) / -2$	$2+7 x (-2)$	$(-3)^2$
E	D	T	H	E	L
$-3 x -8 / -6$	$2 x -3 + (-3)^2$	$-4(-2+5)$	$3-11 / -4$	$(-6)^2 / -4$	$20 - (-2x-11)$
I	F	E	O	U	T
$-2 + 3 x (-4) / 7$	$-7-8 / -2 -3$	$(-4) 3 / (-4)^2$	$-10 + - 16 + -2$		
O	F	I	T		



LEVEL 2

Workshop Title: Equations & Calculations

CODE: L2.WA1.N2

LEVEL 2

WORK AREA 1

PARTNER: FEH

Duration: 7 hours

Learning Outcomes Covered

WORK AREA 1: NUMBERS

Unit 1.3: Solving easy equations

LO1: Solve equations with addition or subtraction.

LO2: Solve equations with multiplication or division.

LO3: Solve simple equations with various operations.

LO4: Use equations to solve simple text tasks.

Unit 1.4: More advanced calculations on integers

LO1: Define rules concerning the sequence of operations.

LO2: Commutativity and associativity of addition and multiplication (operations with brackets).

LO3: Distributivity of multiplication towards addition (operations with brackets). These are level2

LO4: Make complex operations.

LO5: Calculate squares of natural numbers (e.g. area of the square, area of the rectangle).

LO6: Make simple calculations related to the clock and calendar.

Short introduction to the workshop

This workshop includes 5 activities. They collect some previous knowledge, which must be explained by the tutor in order to successfully carry out the activities, such as distributive of multiplication towards addition and solve simple equations with various operations.

With the proposed exercises the resolution of simple equations by students will be enhanced, with visual support and dynamic games, the mentor has a disposition. Basic numerical skills for geometric visual interpretation will also be developed. So useful skills in everyday life, such as the interpretation of the time in analog or digital, or read a text with mathematical meaning.

At a more advanced level, geometric area analysis games are proposed.

How the workshop (its activities) teach the LOs to the participants

First of all, before beginning to practice the problems of equations we must take into account a series of tips that will be useful to us. **To solve the problems of equations we must before starting, make a thorough reading of it.** Getting familiar with the problem is key before you start. Once we have understood the context and the type of problems of equations that are posed to us, we must carry out its approach. If necessary, we will make a drawing, a table, or a representation of the above. Once done, try to identify the unknown and the data provided by the problem. To raise the equation we will return to the problem and we must "translate" it into an algebraic expression. The next step is to solve the equation. Lastly and very important, is to interpret the solution. Always, always, we must detect that our solution is consistent with the above. The translation we have made of our equation problems must be logical and accurate. The following activities shall meet the Learning Objectives.

- **Mathematical Bingo of Equations.**
- **Domino of equations.** The objective of the game is to place all your chips on the table before the opponents and score points. The player who wins a round adds points according to the chips that the opponents could not place.
- **The Mathematical Gymkhana.** Translation from ordinary language to algebraic language. Develop the ability to understand a sentence in natural language, and have the ability to translate everyday phrases into mathematical expressions.
- **The Square.** This puzzle is known as Pythagoras. It was first produced at the end of the 19th century by F.A. Richter and Company. It develops visual perception as the mathematical and geometric logic of the students.
- **Dominoes geometric figures.** Students must join the cards depending on the area of the figures that appear. It implies the calculation of areas, such as visual perception.

Challenges during the implementation of activities and how can be addressed

Lack of understanding of some of the games/plays and the rules that are part of the learning process. Reluctant to participate in the game out of shyness or a degree of difficulty unmotivating. If required, trainers should check online the extra resources included.



ACTIVITY 1: MATHEMATICAL BINGO OF EQUATIONS

Game rules:

Step 1 The Game is for the entire class group.

Step 2 A bingo card is distributed per student.

Step 3 A person is appointed to lead the game (may be the teacher)

Step 4 The person in charge of the game has the students draw successively with the equations and without replacing their cards.

Step 5 Each time a card is drawn, write the equations to be solved on the board, leaving time between one equation and another.

Step 6 Students indicate on their BINGO cards the solutions to the equations they obtain.

Resources for Trainer

- Annex 1

Other Resources needed

- Annex 1 (printed, one per person)
- A Training Room
- Necessary material.
- 20 Cards. Each card has an equation that is considered Level 2 by having denominators. The solutions of all the equations are the values 1, 2... up to 20.
- Bingo cards, one for each student.
- An alternative is to give students empty tables of, for example, 9 boxes that they must fill in (with a pen) with the nine numbers they want between 1 and 20 taken only once. For example, a student can choose these nine values before starting the game. Annex 1.2

ACTIVITY 2: DOMINO OF EQUATIONS

To play dominoes, 28 rectangular tiles are necessary. Each token is divided into 2 equal spaces in which a figure from 0 to 6 appears. The tokens cover all possible combinations with these numbers. It can be played with 2, 3 or 4 players or in pairs..

Step 1 In its turn, each player must place one of their tokens on one of the 2 open ends, so that the points on one side of the chip match the points on the end where it is being placed (In this case the result of the equation will be taken into account, the results must be compatible).

Step 2 Once the player has placed the chip in its place, its turn ends and passes to the next player. The doubles are placed transversely to facilitate their location.

Step 3 If a player cannot play, he must "steal" as many chips as necessary from the pot.

Step 4 If there are no chips left in the pot, it will be the turn of the next player.

Step 5 The game ends when a player or couple reaches the amount of points indicated in the table options.

Resources for Trainer

- Annex 2

Other Resources needed

- Annex 2 (printed, one per group)
- A Training Room
- Necessary material:
- To play dominoes, 28 rectangular tiles are necessary. Each token is divided into 2 equal spaces in which a figure from 0 to 6 appears. The tokens cover all possible combinations with these numbers.



ACTIVITY 3: THE MATHEMATICAL GYMKHANA.

This exercise focuses on the translation from ordinary language to algebraic language. It is a game for four, five or six players. It can be played individually or in teams of two.

Step 1. Five cards are distributed to each team ([Annex 3](#))

Step 2 Each team is given a sheet with the table of sentences.

Step 3 Each team must first translate the phrases into their symbolic expression, simplify the expressions to the maximum, and then solve the questions that appear on their five cards.

Step 4 Win the team that finishes first and correctly solves all five cards.

Resources for Trainer

- Annex 3

Other Resources needed

- Annex 3 (printed, one per group)
- A Training Room
- Necessary material:
 - 15 cards
 - The table with the phrases

ACTIVITY 4: THE SQUARE.

This puzzle is known as Pythagoras. It was first produced at the end of the 19th century by F.A. Richter and Company

Step 1 Arrange all the pieces of the figure so that you form a square. (It can be individual or in groups)

Step 2 The pieces can be rotated.

Step 3 The first who forms the square wins.

Resources for Trainer

- Annex 4

Other Resources needed

- Annex 4 (printed, one per group)
- A Training Room
- Necessary material:
- Print the pieces
- trim them



ACTIVITY 5: DOMINOES GEOMETRIC FIGURES.

Students must join the cards depending on the area of the figures that appear

To play dominoes, 28 rectangular tiles are necessary. Each token is divided into 2 equal spaces in which a figure from 0 to 6 appears. The tokens cover all possible combinations with these numbers.

It can be played with 2, 3 or 4 players or in pairs. The objective of the game is to place all your chips on the table before the opponents and score points. The player who wins a round adds points according to the chips that the opponents could not place.

Step 1 In its turn, each player must place one of their chips on one of the 2 open ends, so that the points on one side of the chip match the points on the end where it is being placed (In this case the result of the geometric area will be taken into account, the results must be compatible).

Step 2 Once the player has placed the chip in its place, its turn ends and passes to the next player. The doubles are placed transversely to facilitate their location.

Step 3 If a player cannot play, he must "steal" as many chips as necessary from the pot.

Step 4 If there are no chips left in the pot, it will be the turn of the next player.

Step 5 The game ends when a player or couple reaches the amount of points indicated in the table options.

Resources for Trainer

- Annex 5

Other Resources needed

- Annex 5 (printed, one per group)
- A Training Room
- Necessary material:
- Print the pieces
- trim them



ANNEXES

Annex 1

$32:(1-5)+27/3$	$4:(3-1)+5(2-4)+10$	$7(3-4)+30:3$	$8-4(3-2)$
$5-3(5-2)+9$	$13+3(4-8)+25:5$	$20:(1-5)-6(1-3)$	$-21:(2-5)+1$
$3+4-2(1-3)-7$	$2-4(7-9)$	$5-3(2-4)$	$-35:(5-10)+5$
$2(5-7)+3+5-(-2)$	$3+44:(7-3)$	$3(7-8)-6+48:2$	$4-6(5-7)$
$(-3)(7-9)-5(-2)+1$	$8-(11-15)-3(-2)$	$(9-4)-2(-7)$	$(-7)(3-5)+6$



ANNEX 1.2

1	3	5
8	12	6
9	11	7



Annex 2

$x-2=1$ ● $x-4=6$	0 ● $3x=9$	-2 ● $x+6=4$	2 ● $x+2=2x$
$-2x=4$ ● $x-3=3$	$2 \cdot 2$ ● $2x=4$	$3x-7=5$ ● $x-2=2$	$4: (-2)$ ● $x-3=1$
4 ● $x-4=4-x$	-3 ● $x+9=13$	-3 ● $5-x=2$	$4-4$ ● $x-x=x$
$x+2=-1$ ● $3x=0$	$2x+4=0$ ● $-4x=-8$	$5x=15$ ● $2x-3=3$	$6-2x=0$ ● $x-6=-2$
$1-x=-1$ ● $6-x=6$	$2x+6=0$ ● $4x-12=0$	$x+3=1$ ● $2x=6$	$2x-1=3$ ● $x+5=2$
-1 ● $2x=-2$	$3x-6=-3$ ● $2-x=4$	$x+1=0$ ● $x+4=8$	$6-7$ ● $2x=x$
$3=x-4$ ● $4x=-8$	$1-x=2$ ● $9+3x=0$	$3x-3=0$ ● $x-3=0$	$x+3=5$ ● 3

Annex 3

twenty-nine times a number ____ $29n$

a number added to fifty-six ____ $56 + n$

the sum of seventeen and a number ____ $17 + n$

a number increased by three ____ $n + 3$

the difference between seventy five and a number ____ $75 - n$

a number increased by fifty-eight ____ $n + 58$

eight less than a number ____ $n - 8$

the difference between nineteen and a number ____ $19 - n$

ten more than a number ____ $n + 10$

eighty nine more than a number ____ $n + 89$

the quotient of a number and ninety-five ____ $n / 95$

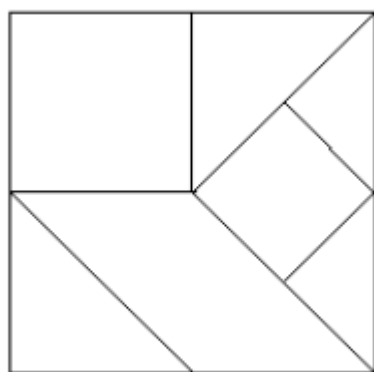
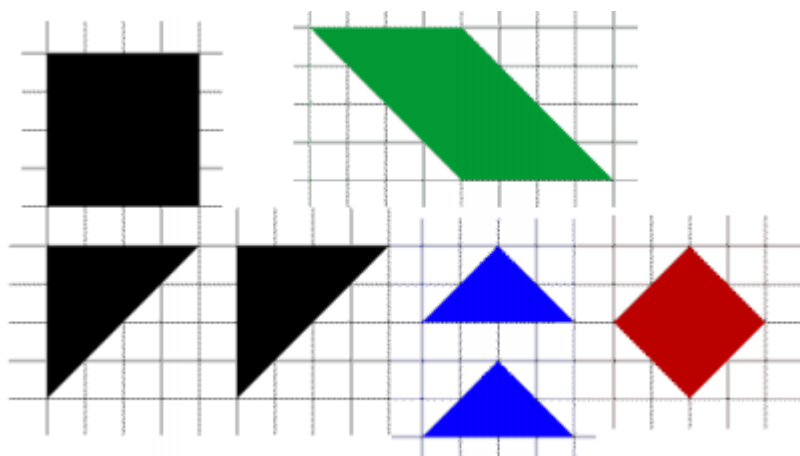
the sum of a number and sixty-six ____ $n + 66$

the quotient of a number and sixty-five ____ $n / 65$

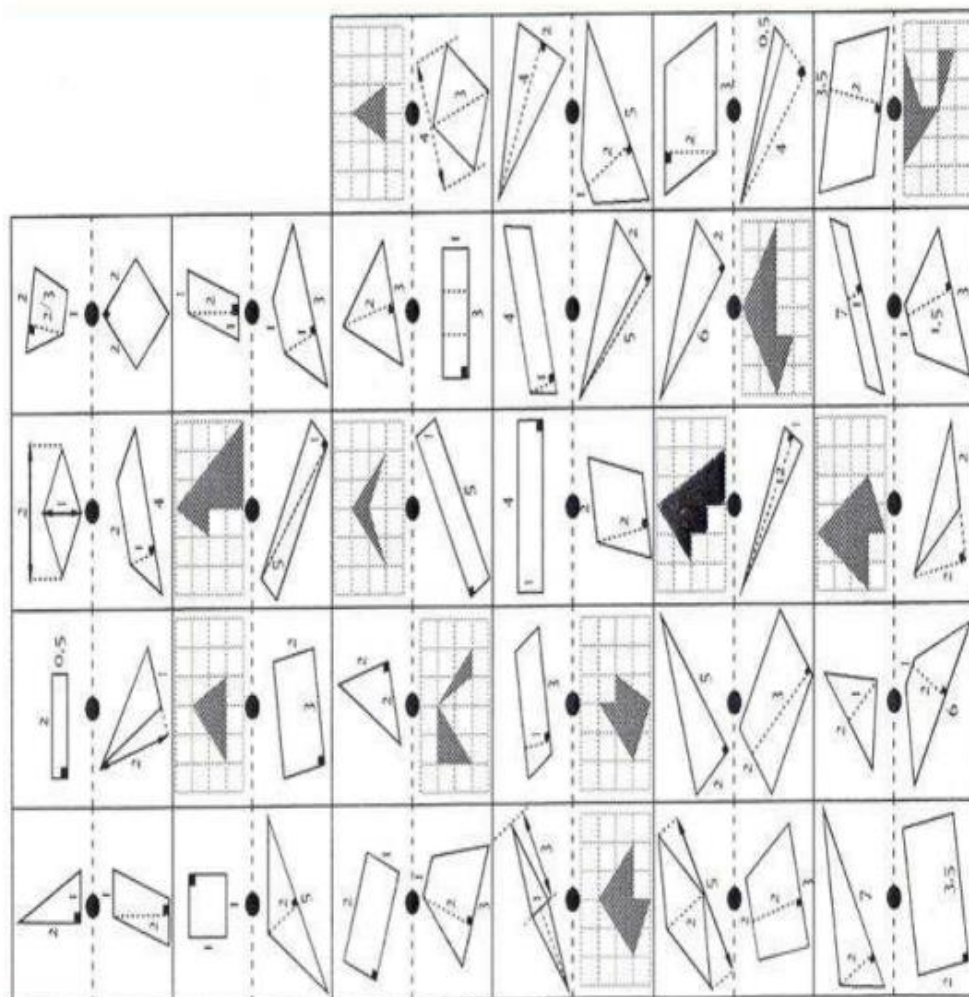
fifty-one times a number ____ $51n$

the sum of twenty two and a number ____ $22 + n$

Annex 4



Annex 5



WORK AREA 2: MEASUREMENT &
CONVERSIONS

LEVEL 2

Workshop Title: Metric Units of measurement and conversions (Part 1)

CODE: L2.WA2.N3

LEVEL 2

WORK AREA 2: MEASUREMENT & CONVERSIONS

Partner: ERIFO

Learning Outcomes Covered

Duration: 4,30-5 hours

Work Area 2: MEASUREMENT & CONVERSIONS

This WS address the LOs of the Work Area 2 referred to the AC ladder level 2.

In particular, the metric units of measurements (Unit 2.1), dates and times (Unit 2.4) and solving problems with measurements and conversion (Unit 2.5)

Unit 2.1 Metric Units of Measurement

LO1: Understand and List metric units of length and distance (used in the local, national context). (km, m, cm, mm) & (miles)

LO2: Understand and List the metric units for area. (cm², m², km²)

LO3: Understand and List metric units for volume. (m³, cm³, mm³, l, ml)

LO4: Understand and List units of time (year, month, day) & (hr, m, s)

LO5: Understand and List units for angle measurement (degrees)

Unit 2.4: dates and times

LO1: Read dates in various formats (day/month/year) or (month/date/year)

LO2: Write dates in various formats (day/month/year) or (month/date/year)

LO3: Organize dates chronologically

LO4: Demonstrate date calculations (future and past)

Unit 2.5 Solving simple problems with measurements and conversions

LO1: Explain a watch with watch hand



Short introduction to the workshop

The Workshop is structured into 4 activities in order to provide specific knowledge and skills and help low-skilled adults to achieve the level 2 of the AC ladder.

The activities will be the following ones:

Activity 1: dates and chronological timeline of the events

Activity 2: angles measurement (angle degree) and watch

Activity 3: length and distance

Activity 4: area and volume

How the workshop (its activities) teach the LOs to the participants

The approach on which the workshop is based is experiential therefore activities refer to real-life situations and examples of daily life tasks. They will become aware of how numeracy skills can help them carrying out these tasks. The experiential approach is particularly effective as students are asked to deal with challenges they may have already encountered in their daily life and thus they will become quickly aware of the importance of numeracy skills and be more motivated to learn. Experiential learning focuses on learners reflecting on their experience of doing something, so as to gain conceptual insight as well as practical expertise. This approach will be fundamental to motivate learners and ensure the achievement of the final objective of the workshops: the development of the knowledge and skills needed in today's society and particularly in the world of work.

Challenges during the implementation of activities and how can be addressed

This Workshop will be developed as an interactive steps game which foresees the active involvement of participant. The diversification of the activities will guarantee major possibility to gain the learners attention and interest. Moreover, the WS is developed to permit them to find easily connections with situations that they tackle every day.

Learners may feel discouraged if they are not able to carry out a specific task. The mentor has to promptly support and motivate them by providing further explanations and clarifications or, if needed, showing how the work has to be done, step-by-step. Moreover, the experiential approach and constant referral to real-life situations will be fundamental to keep learners' attention and motivation high.



Step by step instructions for the trainers/mentors

Activity 1: When did it happen?

Step 1: the mentor has to provide a printed timeline ([Annex 1](#)) in which learners have to indicate some important events of their own life. For example, the birthday, the wedding date, childbirth, etc. The dates should be written in various formats (day/month/year) or (month/date/year).

Step 2: the learners have to identify the chronological timeline of the events: past, present, future (they have to imagine something that will happen in their life in forthcoming future)

Step 3: the mentor has to provide a second printed timeline in which learners have to identify the public holidays (e.g. 1st of May, Wednesday, Labour Day).

Step 4: Learners have to identify the chronological timeline of the public holidays (e.g. "It is 20th December. So just 4 days until Christmas.")

Step 5: the mentor has to provide a facsimile of a logbook ([Annex 2](#)) and asks learners to fill it in with hypothetical daily activities, beside specifying the exact time (e.g. going to supermarket, 18/01, hr: 9.30 AM)

Resources for Trainer

- Printable timeline
- Logbook facsimile

Other Resources needed

- Training room

ACTIVITY 2: What time is it? It's 90° degrees to midday

Step 1: The mentor explains time and angles using a watch; then he/she provides learners a printed kit composed by a circle, two straight lines and a pin ([Annex 3](#)).

Step 2: The learners have to build a paper watch using the printed kit.

Step 3: Playing with the watch by moving its hands, the learners have to identify different types of angles on the watch and the time that corresponds to each identified angle. For example, at 9 o' clock it's a right angle

Step 4: the mentor has to provide a printed goniometer (<https://artechpro.com/images/templates/G1.pdf>) and he/she has to explain its use.

Step 5: The learners have to overlap the printed goniometer on the printed watch and measure the amplitude of the different angles formed by the watch hands

Step 6: The previous steps are preparatory for filling a Questionnaire, which will test the ability of learners to measure angles; it will include several illustrations representing angles and ask the learner to define the exact amplitude (e.g. 135°) and the type of angle (e.g. obtuse°). The questionnaire will also include questions referring to time that learners have to fill in.

Resources for Trainer

- Printable kit to create a watch
- Printable goniometer

Other Resources needed

- Outside space
- Training room

<https://artechpro.com/images/templates/G1.pdf>



ACTIVITY 3: How long is it?

Step 1: Overview about the metric measures of length (Millimeters, Centimeters, Meters, Kilometers, etc.) and their symbols (mm, cm, m, km, etc.), using illustrated cards (see example in [Annex 4](#)).

Step 2: Learners have to go outside with a measuring tape and sign on a chart several distance and length measurements

Step 3: Evaluation phase by comparing the measurement done by each learner.

Resources for Trainer

- Measuring tape
- Working sheets

Other Resources needed

- Outside space
- Training room



ACTIVITY 4: How wide is it?

Step 1: Overview about the metric measures of area and volume and their symbols (cm^2 , m^2 , km^2 ; m^3 , cm^3 , mm^3) thanks to illustrated cards (see example in Annex 4). The trainer will show learners the different formula to calculate area and volume

Step 2: Using the measurement tape, the learners have to take measures of different objects or places (like the room in which they are attending the WS).

Step 3: Helped by the mentor, they have to cooperate in order to recognize area and volumes of the aforementioned objects.

Resources for Trainer

- Illustrated cards (length)
- Illustrated cards (area)
- Illustrated cards (volume)
- Measuring tape

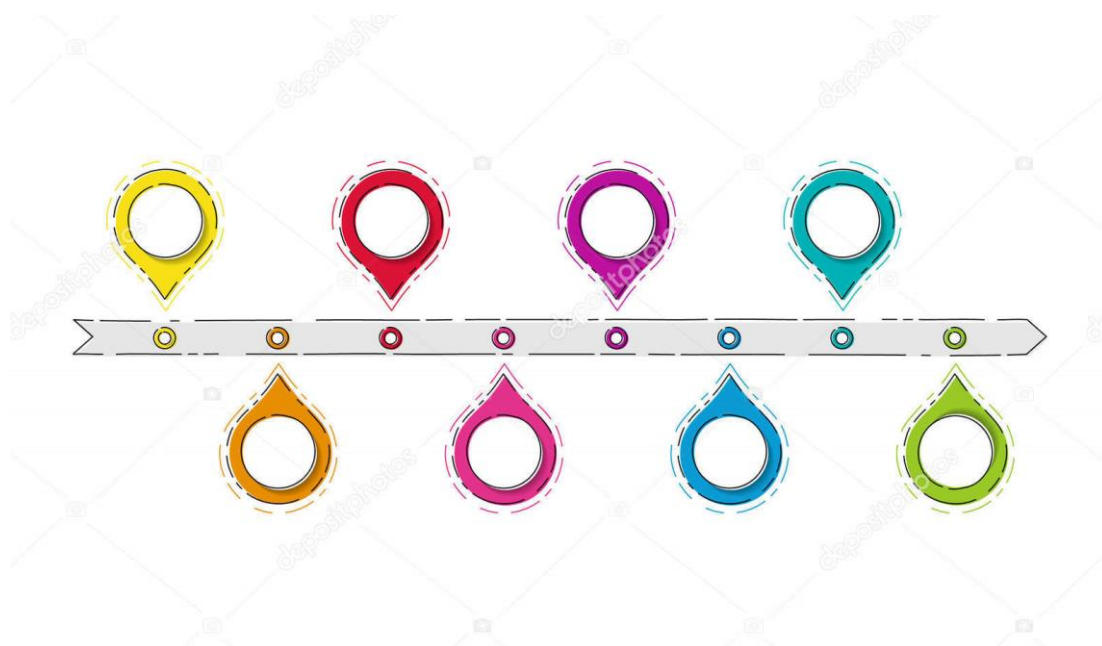
Other Resources needed

- Outside space
- Training room
- Working Sheets
- <https://artechpro.com/images/templates/G1.pdf>



ANNEXES

Annex 1



Annex 2

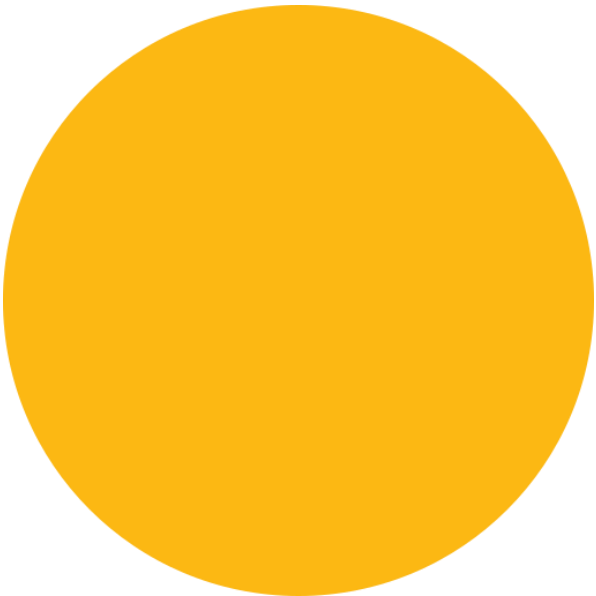
WEEK _____

DAILY SCHEDULE

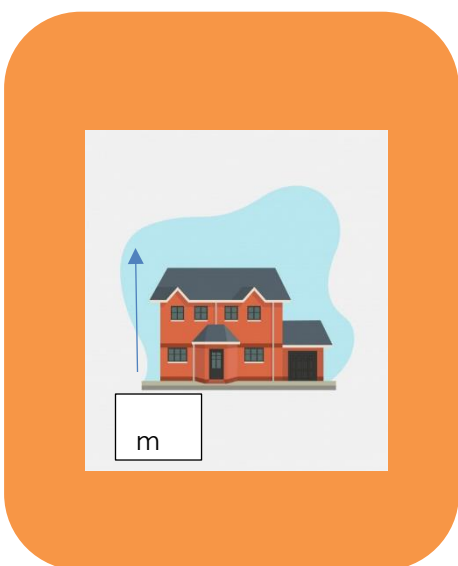
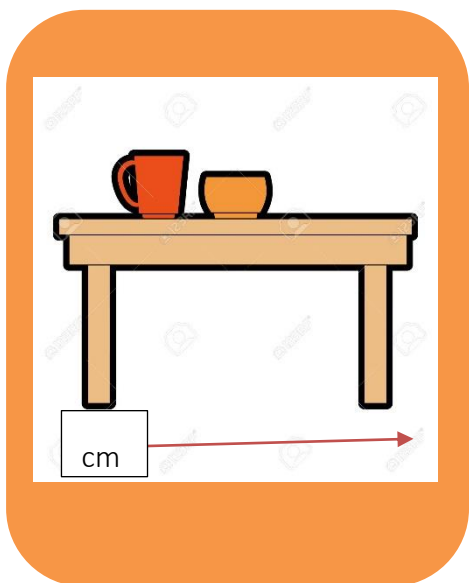
TIME	MON	TUE	WED	THU	FRI	SAT	SUN
5:00 A.M.							
5:30 A.M.							
6:00 A.M.							
6:30 A.M.							
7:00 A.M.							
7:30 A.M.							
8:00 A.M.							
8:30 A.M.							
9:00 A.M.							
9:30 A.M.							
10:00 A.M.							
10:30 A.M.							
11:00 A.M.							
11:30 A.M.							
12:00 P.M.							
12:30 P.M.							
1:00 P.M.							
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4:30 P.M.							
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6:30 P.M.							
7:00 P.M.							
7:30 P.M.							
8:00 P.M.							
8:30 P.M.							
9:00 P.M.							
9:30 P.M.							
10:00 P.M.							



Annex 3



Annex 4



LEVEL 2

Workshop Title: Square has a dream about a visit to a one - dimensional world
CODE: L2.WA3.N4
LEVEL 2
WORK AREA 3: SHAPES & SPACE
Partner: DIMITRA
Learning Outcomes Covered:
Duration: 5 - 7 hours
WORK AREA 3: SHAPES & SPACE Unit 3.1: Basic Geometric Shapes LO1: Identify the points, lines and angles LO2: Illustrate by example parallel and perpendicular lines LO3: Identify the basic 2D geometric shapes - Square, Rectangle, Triangle, Circle, Rhombus LO4: Draw basic 2D geometric shapes LO5: Describe the basic characteristics of each one of the basic shapes LO6: Measure angles, sites, diameter, radius. LO7: Identify the 3D geometric shapes (Sphere, Cone, Cube, Cylinder, Prism, Pyramid) LO8: Draw basic 3D geometric shapes LO9: Solving a basic geometric problem in everyday settings
Short introduction to the workshop
The workshop includes learning activities that refer to the basic Geometric shapes. The content of these activities focuses on the presentation of basic shapes and basic points, lines and angles and provide guidance to fully conceive and distinguish them. In addition, through this workshop the trainee will be able to describe the basic characteristics of each shape and measure: • angles, • sites, • diameter, • radius.



How the workshop (its activities) teach the LOs to the participants

Through the use of things that are familiar to the trainees and their everyday life, such as art pieces, common used objects etc, the trainees will be able to recognize easier the different shapes, distinguish their characteristics and understand in a better way their use and measurement.

All these learning outcomes will be achieved both through theoretical presentations and practical activities for better conception of the learning objective.

Challenges during the implementation of activities and how can be addressed

During the implementation of activities, the main challenge that might be faced is the difficulty in the measurement of some characteristics of shapes. This problem can be moderated by working in a more practical level than that of a theoretical basis, in a sense that through extensive practical exercises the trainees will become easier familiarized with the shape measurement procedures.



Step by step instructions for the trainers/mentors

ACTIVITY 1: The shape of Art

Step1: Short presentation of basic Shapes.

Step 2: Use of images of everyday items that are shaped according to these basic shapes and ask them to identify the shapes.

Step 3: Different Art pieces to which you may recognize different shapes.

Step 4: Draw the shapes that they might find in the pictures.

Step 5: Give them materials so they can turn some of the 2D materials in 3D (ie. paper to create a sphere, straws to create a pyramid or a cube, legos for cubes etc.)

Step 6: Everybody presents what they did and describe the whole process.

Resources for trainer:

- Khan academy, Recognizing shapes,
- https://www.youtube.com/watch?v=10dTz1Zy_4w (accessed on 19/9/2019)
- *Khan academy, Recognizing common 3D shapes,*
<https://www.youtube.com/watch?v=tqxQSSzuXX0> (accessed on 19/9/2019)
- Slideshare Net, Math Geometry Guide,
<https://www.slideshare.net/melissaramjattan1/2d-3d-shapes-sing-along-pp>
https://www.slideshare.net/guestb6058c/math-geometry-guide?qid=e35f92a2-bc05-4efb-a89c-ce08684c3d50&v=&b=&from_search=1
(accessed on 20/9/2019)

Other Resources:

- A training room or an outdoor space.
- Different art pieces, straws, paper, tape, paper, measuring tools
Fruits containers, fruits, gloves, knives, paper towel.



ACTIVITY 2: *Shape*playing in the room

Step 1: Short presentation of basic points, lines and angles and how to measure those.

Step2: Let them find one example of each in the room or in your staff.

Step 3: Ask them to measure length, radius, diameter etc. and ask them to write it on a paper.

Step 4: Give them the material of the previous exercise and ask them to recreate the shape in a scale that you will name (ie. 1/10)

Step 5: Let them present this and the process.

Resources for trainer:

- Mr. Pearson (channel), Points Lines Rays,
<https://www.youtube.com/watch?v=9dsmVP82tig> (*accessed on 19/9/2019*)
- Slideshare Net, Math Geometry Guide,
<https://www.slideshare.net/melissaramjattan1/2d-3d-shapes-sing-along-pp>
https://www.slideshare.net/guestb6058c/math-geometry-guide?qid=e35f92a2-bc05-4efb-a89c-ce08684c3d50&v=&b=&from_search=1
(*accessed on 20/9/2019*)

Other Resources:

- A training room or an outdoor space.
- Different art pieces, straws, paper, tape, paper, measuring tools
- Fruits containers, fruits, gloves, knives, paper towel.



ACTIVITY 3: Fruit pick nick!

Step 1: Bring some containers and fruit to the classroom and ask them to cut those in various shapes in order for them to fill in the containers with at least three different fruit (make sure that they understand they need to feel the container with as much quantity as possible). Ask them to explain why they did it this way, which shapes they picked and give them a minute to think what they would have done differently if they wanted to waste less fruit (connect it to the next workshop)

Step 2: Enjoy the fruits ☺ !

Resources for Trainer:

None

Other Resources:

Basket of Fruits



LEVEL 3

WORK AREA 1: NUMBERS

Workshop Title: Factors and rules of order in calculation

Duration: 7 hours

CODE: L3.WA1.N1

LEVEL 3

WORK AREA 1: NUMBERS

PARTNER: FEH

Learning Outcomes Covered

WORK AREA 1: NUMBERS

Unit 1.9 Sequences

Define a factor

LO1: Demonstrate that a number is a factor of another by splitting a number into its factors

LO2: List the factors of a number

LO3: Break a number down to its prime factor (Prime factorization)

LO4: Find the greatest common factor of a set of number

LO5: Define a multiple

LO6: Generate a list of multiples (multiplication table)

LO7: Find the least common multiple

LO8: Use prime factorization to find the least common multiple

LO9: Remember square numbers sequence (from 1 to 12)

Unit 1.10 Rules of Order in Calculations

LO1 Being able to recognize rules of order

LO2 Parentheses

LO3 Evaluate the multiplication and division from left to right.

LO4 Evaluate the addition and subtraction from left to right.

Short introduction to the workshop

In math problems it's important to do the operations in the right order. If you don't, you may end up with the wrong answer. In math, there can be only one correct answer, so mathematicians came up with rules to follow so we can all come up with the same correct answer. The correct order in math is called the "order of operations". The basic idea is that you do some things, like multiplication, before others, like addition.

Here you will find information and interactive videos on how to use the rules in parentheses:

<https://study.com/academy/lesson/using-parentheses-in-math-rules-examples.html>

Within this link, you can also find a complete factorization course:

<https://www.mathhelp.com/accuplacer-math-test-prep/?lsn=Prime+Factorization>

How the workshop (its activities) teach the LOs to the participants

With these activities it is sought that students become familiar and develop skills in the identification of notable products and factoring in an environment of trust, freedom and cooperation.

Polynomial soup. This game is designed to play from one to four players, and each group must have a board and sixteen cards with polynomials like the ones below.

Magical Squares. Less known than normal magic squares, there are also other types of magic squares, multiplicative magic squares, those in which the product of all the numbers of each row, of each column or of the diagonals always gives the same.

Building a Pitágoras Puzzle. We present a beautiful activity to be done with any geometry program such as Geogebra or if you can't, with traditional drawing instruments. In the presentation to the students of the activity, the method used to do the construction is not specified, leaving that, at the discretion of the classroom teachers. Students will easily build a puzzle that allows them to prove Pythagoras' theorem. Hence the name of this entry.

Domino of products with whole. The rule of the signs. This domino to multiply whole numbers and strengthen the sign rule. Playing this game, it is intended that students reinforce the rule of signs, multiplying two integers with each other. This is a game to use when the concept of whole numbers has just been introduced, when students still have to handle the notation of integers with parentheses such as $(+2)$ or (-3) .



How to factor

When we talk about factoring, we can follow the following recommendations:

1. Observe if there is a common factor, that is, if there is a factor that is repeated in the different terms.
2. Order the expression: sometimes when we fix the expression we realize the possibilities of factoring.
3. Find out if the expression is factorizable: sometimes we are in the presence of expressions that cannot be broken down into factors.
4. Verify if the factors found are in turn factorizable.

ACTIVITY 1: POLYNOMIC SOUP

General remarks

1. Type Board: Numerical-algebraic
2. Number of players: Four
3. Objectives: Practice factoring

The objectives that we intend with this game are the following:

1) Factor polynomial.

- 1) Check that there are polynomials that cannot be fully factored into factors of degree 1, reasoning why.
- 2) Work the mental calculation.
- 3) Work the root relation (solution or zero) of a polynomial with that of factor and vice versa.
- 4) Solve equations.

Game rules:

Step 1: The 16 cards are shuffled and placed face down on the table and each player, in turn, chooses a card to total four of them. (Annex 1)

Step 2: Players factor their polynomials, and look for, in the soup of factors that appears on the board, the consecutive factors of each factorization and they mark them.

Step 3: The player who manages to mark the decompositions of his four polynomials first, in a time established beforehand. If no one has succeeded, the one with the most polynomials to do loses.

Resources for Trainer

- Annex 1,

Other Resources needed

- Annex 1
- A Training Room
- A White Board and markers
- Material Needed Board and cards



ACTIVITY 2: MAGICAL SQUARES

Objectives: Strengthen the factorization of simple numbers. / Practice operations with natural powers. Here you have a multiplicative magic square where some numbers are missing in the boxes: (See annex 2)

Step 1. Factor all the numbers that appear, and find in a factored form the magic number of the square.

Step 2. Thanks to this magic number and remembering the properties of the powers, you can also obtain in a factored form the numbers of the blank boxes.

Step 3. Then calculate all those numbers based on their factors.

Resources for Trainer

- Annex 2

Other Resources needed

- Annex 2
- A Training Room
- A White Board and markers
- Print the boxes



ACTIVITY 3: BUILDING A PITÁGORAS PUZZLE

This activity helps the student to reinforce their perception and geometric reasoning

Pythagoras 'Puzzle 'consists of a board with a diagram on it and seven puzzle pieces to place on it. The diagram shows a right triangle (in white) with red, yellow, and blue squares constructed on the three sides. According to Pythagoras' theorem, the area of the blue square is equal to the sum of the areas of the red and yellow squares. to demonstrate this, the red puzzle pieces can be used to make a red square and the yellow puzzle pieces can be used to make a yellow square, and then they can be turned over and used to make a blue square.

In the picture at right the puzzle parts are not turned over, which makes the point that the red square reassembles into a red rectangle and the yellow square reassembles into a yellow rectangle (the two blue pieces in the picture above are red on the other side). Many proofs of Pythagoras' theorem use this feature. Also note that in reassembling the squares the parts were moved but keep their same orientation. [Annex 3.1](#)

This isn't a complete proof of Pythagoras' theorem, because it isn't clear that the parts exactly cover the squares (in fact, they don't, because they haven't been cut accurately), and because the rule used for cutting up the squares doesn't work for all right triangles (but it does work when none of the angles are smaller than 20 degrees).

Step 1: Draw two contiguous squares like the ones in the figure, the largest with 8 cm sides and the smallest with 6 cm. that results in a triangle. [\(See annex 3.1\)](#)

Step 2: Next, draw the following two segments [\(See Annex 3\)](#)

Step 3 Cut out the 12 pieces thus obtained and try to build a large square with some of them. Answer the question: What are the sides of this great square?

Step 4 Justify with your words that it is a geometric proof of the Pythagorean theorem

Resources for Trainer

- Annexes 3

Other Resources needed

- Annex 3
- A Training Room
- A White Board and markers
- Print the square



Activity 4: DOMINO OF THE PRODUCT WITH WHOLE: The rule of the signs

Observations:

The structure of the classic dominoes, 8 times the 0, 8 times the 1, etc., up to 8 times the 6, obtaining the 28 dominoes by means of all the possible combinations of 7 results, taken two by two, plus the seven doubles tokens, has been reproduced in the 28 tokens we present, changing the numbers of a classic domino by integers added together.

The rules of the game are exactly the same as those of the usual dominoes.

The 7 values that have been used as integers for the 28 tabs are as follows:

0 (+24) (-24) (+36) (-36) (+48) (-48)

Exercise

It is about playing some domino games with these 28 chips, in the same way exactly that is played with the traditional dominoes.

For that, the cards can be photocopied, enlarging them, in a cardboard that will be plasticized so that it has a sufficiently hard consistency and so that it can be used on subsequent occasions. Then the plasticized chips will be trimmed.

In a normal class session you can play several games, for example a tournament in the class group, as explained on the page of this blog dedicated to DOMINOS

Game rules:

Game for two or four players.

Step 1 7 chips are distributed per player. If they are two players, the remaining chips remain on the table face down to be picked up at the time.

Step 2 The player with the white double token comes out.

Step 3 Players must place their linked tiles with the first on each side of the tile, using numbers with the same value.



Step 4 If a player cannot place a chip because he does not have adequate values, he loses his turn. In the case of two players, take a new chip until you get the right one or use them out.

Step 5 The player who runs out of chips wins. If the game is closed and no one can place a chip, the player with the least points wins, adding the values of the chips he has left.

Resources for Trainer

- Annex 4

Other Resources needed

- Annex 4
- A Training Room
- A White Board and markers
- Print the tokens



ANNEXES

Annex 1

$x-1$	$x+1$	$x-2$	$2x+3$	$1-x$
$x-1$	x	$x-7$	$x-2$	$x+4$
$x+2$	$5x+2$	$x+3$	$x+1$	$x-2$
$x+6$	x	x^2+1	$3x-2$	$2x^2+1$
$3x^2+2$	x	$-2x-1$	$x+1$	$-x^2-1$
$x-3$	$4x-1$	$x+2$	$x-2$	$3-x$

1 x^3-2x^2-x+2	2 x^3+3x^2+x+3	3 $2x^3+x^2-7x-6$	4 x^3-3x+2
5 x^3+2x^2-3x	6 $6x^3-4x^2+3x-2$	7 $-x^3+7x-6$	8 $x^3-6x^2+12x-8$
9 $4x^3-x^2$	10 $5x^3+7x^2+2x$	11 $-2x^3-5x^2-2x$	12 $-2x^3-5x^2-23x+6$
13 $3x^3-9x^2+2x-6$	14 $-x^3+3x^2+4x-12$	15 $3x^3-5x^2-4x+4$	16 x^3+x



Annex 2

12	36	14	5	10
35	2		16	6
1		15		20
	7	4	30	
			3	

Solution

1. This is the factorization of all the numbers that appear
2. The missing numbers are easy to find if done in order, they are as follows. this implies that the magic square is



SOLUCIÓN

1. Esta es la factorización de todos los números que aparecen:

$2^2 \cdot 3$	$2^2 \cdot 3^2$	$2 \cdot 7$	5	$2 \cdot 5$
$5 \cdot 7$	2		2^4	$2 \cdot 3$
1		$3 \cdot 5$		$2^2 \cdot 5$
	7	2^2	$2 \cdot 3 \cdot 5$	
			3	

 $2^6 \cdot 3^3 \cdot 5^2 \cdot 7 = 302400$

2. Los números que faltan son fáciles de hallar si se procede con orden. Son los siguientes:

$2^2 \cdot 3$	$2^2 \cdot 3^2$	$2 \cdot 7$	5	$2 \cdot 5$
$5 \cdot 7$	2	$3^2 \cdot 5$	2^4	$2 \cdot 3$
1	$2^3 \cdot 3$	$3 \cdot 5$	$2 \cdot 3 \cdot 7$	$2^2 \cdot 5$
$2^3 \cdot 5$	7	2^2	$2 \cdot 3 \cdot 5$	3^2
$2 \cdot 3^2$	5^2	2^3	3	$2^2 \cdot 7$

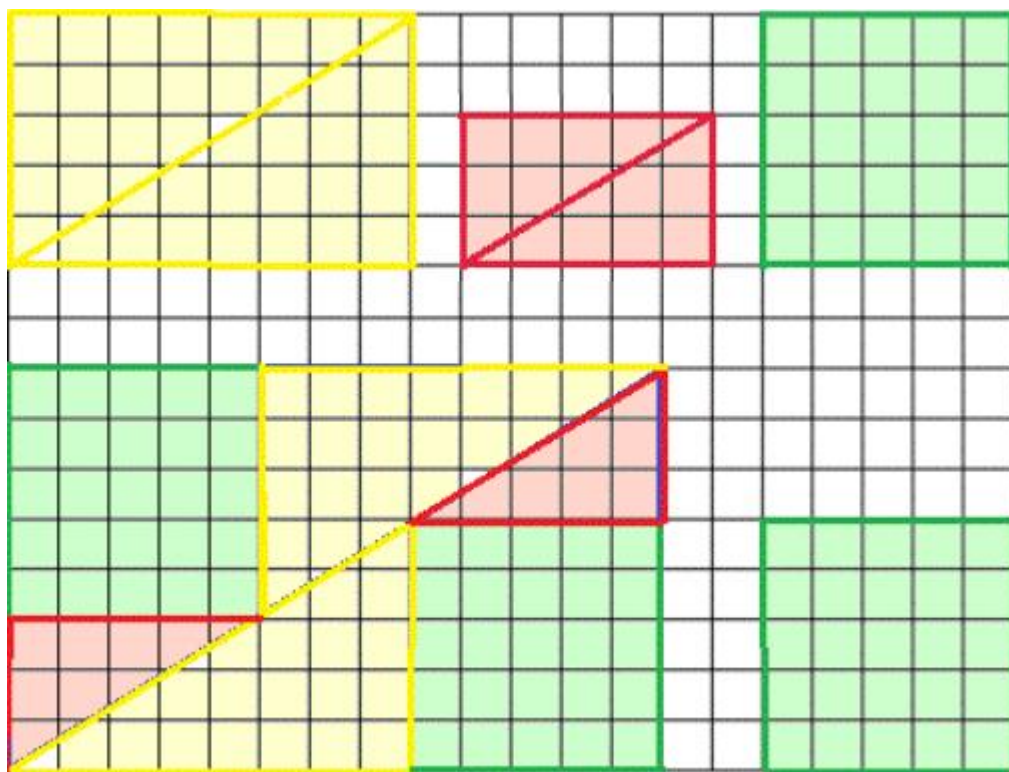


Esto implica que el cuadrado mágico es:

12	36	14	5	10
35	2	45	16	6
1	24	15	42	20
40	7	4	30	9
18	25	8	3	28



Annex 3



3.1 EXAMPLE



Making two squares



Making one square



Annex 4

●	●	●	●	●	●	●
$(-6) \cdot (+8)$	$(-6) \cdot (-8)$	$(-4) \cdot (+9)$	$(+4) \cdot (+9)$	$(-12) \cdot (+2)$	$(-4) \cdot (-6)$	
$(-2) \cdot (-12)$	$(-8) \cdot (-3)$	$(+3) \cdot (+8)$	$(-1) \cdot (-24)$	$(+6) \cdot (+4)$	$(12) \cdot (+2)$	
●	●	●	●	●	●	
$(-24) \cdot (+2)$	$(-24) \cdot (-2)$	$(-18) \cdot (+2)$	$(-18) \cdot (-2)$	$(-3) \cdot (+8)$	$(-8) \cdot (-3)$	
$(-4) \cdot (+6)$	$(+12) \cdot (-2)$	$(-8) \cdot (+3)$	$(+24) \cdot (-1)$	$(-12) \cdot (+2)$		
●	●	●	●	●		
$(+6) \cdot (-8)$	$(+16) \cdot (+3)$	$(-6) \cdot (+6)$	$(+4) \cdot (+9)$	$(-24) \cdot (+1)$		



$(-1) \cdot (-36)$ ●	$(-6) \cdot (-6)$ ●	$(-2) \cdot (-18)$ ●	$(-1) \cdot (-36)$ ●
$(-3) \cdot (+16)$	$(+12) \cdot (+4)$	$(-2) \cdot (+18)$	$(+3) \cdot (+12)$
$(-36) \cdot (+1)$ ●	$(-9) \cdot (+4)$ ●	$(-36) \cdot (+1)$ ●	
$(-48) \cdot (+1)$	$(-4) \cdot (-12)$	$(+4) \cdot (-9)$	
$(-4) \cdot (-12)$ ●	$(-1) \cdot (-48)$ ●		
$(-8) \cdot (+6)$	$(-3) \cdot (-16)$		

$(-16) \cdot (+3)$ ●
$(-1) \cdot (+48)$



LEVEL 3

Workshop Title: Dive in Decimals and Percentages

CODE: L3.WA1.N2

LEVEL 3

WORK AREA 1: NUMBERS

PARTNER: FEH

Duration: 5 hours

Learning Outcomes Covered

Work Area 1: NUMBERS

1.12 Decimals

Understanding the use of decimals

LO1: Explain the role of the decimal point

LO2: Have knowledge of the different meaning of decimals (share, ratio...)

Have knowledge of the names of the numbers before and after the decimal

LO3: Recognize the place value of decimals (Decimal Point, Tenths, Hundredths, Thousandths)

LO4: Read and write numbers written in decimal notation

LO5: Compose and decompose a decimal written in decimal notation

Able to recognize the decimals

LO6: Place the decimals in the right order

LO7: Locate decimals on a number line

LO8: Recognize placeholder zeros

Able to add, subtract, multiply and divide decimals

LO9: Be able to add two or more decimals

LO10: Be able to subtract two or more decimals

LO11: Be able to multiply two decimals

LO12: Be able to divide two decimals



LO13: Approximates (e.g. estimates, rounds to a given value, truncates decimal places)

LO14: Moving the decimal point

Order and compare decimal numbers

LO15: Locate decimals on a number line (between two consecutive natural numbers and between two decimals)

LO16: Compare two decimals

LO17: Arranges decimals in increasing or decreasing order

1.13 Percentage

Being able to calculate percentages

LO1: Calculate the percentage of a specific number

LO2: Add a percentage to a number.

LO3: Subtract a percentage from a number.

Short introduction to the workshop

This workshop includes 4 activities. **Treasure Map:** an activity aims at giving basic notions of multiplication and calculation with decimals by setting a similar game than the search of the treasure. By establishing their own path through the map, several calculations will be made in order to get a final score. Students need to get the highest as possible to win. **Decimal Shopping:** This exercise wants to students to practice additions, subtraction, multiplication and division with decimals in a practical way and in real situations. **Trade your Numbers.** This game aims at helping students understand the place value of decimals, to recognize them, read and write in decimal notation and order and compare decimals with an easy card play. The **Describe your number** activity follows a similar path and aims at helping players and students to arrange decimals in order, compare them and know the place of each part of it (tenths, hundredths, thousandths). Three more activities in the quiz contribute to an overall knowledge of calculate percentages, ordering decimals and subtracting, adding, multiplying and dividing with decimals.

How the workshop (its activities) teach the LOs to the participants

This workshop aims to provide students with a clear understanding of decimals as a whole (recognition, addition, subtraction, order, multiplication, role place value) and its relationship with percentages. It also includes a mixed of activities in a quiz format. A brief explanation follows:

- **Treasure Map:** By establishing their own path through the map, several calculations (multiplication specially) will be made in order to get a final score. Students need to get the highest as possible to win.
- **Decimal Shopping:** Rules of additions, subtraction, multiplication and division with decimals in a practical way and in real situations take place here. Trainers may set different rules so that student practices different rules with multiplication, subtraction or others.
- **Trade your Numbers.** Easy card play that makes student thinks fast about decimals place in order to beat their opponent. Further explanations of the role of decimals might be added.
- **Describe your number** activity follows a similar path and aims at helping players and students to arrange decimals in order, compare them and know the place of each part of it (tenths, hundredths, thousandths). Further types of activities are included so that activities may vary.

Challenges during the implementation of activities and how can be addressed

Lack of motivation. Unwillingness to play games due to shyness of getting involved in contact games with others. Encouragement and willingness to make things easier from the trainer is an asset.



ACTIVITY 1: Treasure Map

This activity aims at giving basic notions of multiplication and calculation with decimals. Students play with the map in **Annex 1**. They begin at the “Start” button with 100 on their calculator. Students must decide which arrows to follow. They can only travel in the direction of the arrows. Teacher may decide which one wins: the one that gets lower or higher points. Teacher may change the rules.

Step1: Provide **Annex 1** to students.

Step 2: Ask students to choose a path through the map. Students have to go through and make the necessary calculations to get to “Finish”.

Step 3: They can split in groups, in pairs or individually. Teacher might explain before the activity takes places, the basic explanations on multiplication on decimals. Information on the “Other resources for Trainer” available.

Step 4: Ask students to explain their priorities and their path through the map. Make corrections and further explanations if needed.

You can check further information on the “*Resources for Trainer*” section.

Resources for Trainer

- Annex 1.
- Multiplication on decimals:
<https://www.khanacademy.org/math/arithmetic/arith-decimals/arith-review-multiplying-decimals/v/intro-to-multiplying-decimals>

Other Resources needed

- A Training Room
- Laptop and video projector possible.
- White board and markers.



ACTIVITY 2: Decimal Shopping!

This exercise wants to students to practice additions, subtraction, multiplication and division with decimals in a practical way and in real situations.

Step1: Split students in pairs, groups or individually, depending on the occasion.

Step 2: Provide to each group/student a catalog of groceries, clothes, furniture or others related that may get their attention.

Step 3: Tell students that they can shop for their own staff but only if they stay within a certain budget. Give them the number that his budget represents. This numbers depends totally on you. It may be 600€, 300€ or other.

Step 4: Set a different rule. Students must choose 5 items and have less than 50€.

Step 5: Set a different rule. Students can have 3 items per product. Explain the type of calculation this is.

Step 6: If they are not in groups, form several groups. Ask them to split products depending on their taste/needs. Explain the basic division rules if necessary. Calculate how much it would cost in the end to each one.

You can check further explanations and examples on the videos available.

Resources for Trainer

- Explanation to add decimals
<https://www.khanacademy.org/math/arithmetic/arith-decimals/arith-review-add-decimals/v/introduction-to-adding-decimals-tenths>
- Explanation of multiplication of decimals:
<https://www.khanacademy.org/math/arithmetic/arith-decimals/arith-review-multiplying-decimals/v/intro-to-multiplying-decimals>
- Explanation of subtracting decimals: <https://www.khanacademy.org/math/cc-fifth-grade-math/subtract-decimals/imp-subtracting-decimals-intro/v/strategies-for-subtracting-basic-decimals>
- Dividing decimals with hundredths
<https://www.khanacademy.org/math/arithmetic/arith-decimals/arith-review-dividing-decimals/v/dividing-decimals-with-hundredths>

Other Resources needed

- A Training Room.
- Paper and pencil for each participant.
- Catalogs of groceries, furniture or clothes (make sure there are decimals in the prices).

ACTIVITY 3: Trade your Numbers

This game aims at helping students understand the place value of decimals, to recognize them, read and write in decimal notation and order and compare decimals. You will need [Annex 2](#) and a deck of cards.

Step1: Each player (student) should have the [Annex 2](#) (scoreboard). You can either form groups, pairs or individuals.

Step 2: Trainer should shuffle the deck and deal 5 cards to each players, placing them down.

Step 3: When the trainer says “Go!” students flip their cards over. Students have to arrange them in order to create the largest decimal number possible. Students have to write this number in the table, in [Annex 2](#).

Step 4: When finished, players should compare their tables. The player who has the largest number wins the game. If results are equal, each player receives a point. Whoever has the highest score after the fifth round wins.

Step 5: Ask students to order the numbers formed. They should take everyone’s players number and place them in order from the lowest to the highest. Explain the differences and basic rules of decimals.

Variations: play the game again. This time two players try to create the lowest number possible.

Resources for Trainer

- Annex 2
- Paper, pencil, deck of cards.

Other Resources needed

- A Training Room
- Laptop and video projector.
- White board and markers.

ACTIVITY 4: Describe your number.

This game aims at helping players and students to arrange decimals in order, compare them and know the place of each part of it (tenths, hundredths, thousandths). It is a cards game. You need a card deck.

Step1: Set your student in pairs. Give **Annex 3** to each student.

Step 2: The cards are shuffled. Ask Player 1 to pick up a card and face it up. Both players must immediately write this number down on any place through Annex 3. Once the number is written down, it cannot be changed to a different place value position.

Step 3: Player 2 draws a card and places it face up. As in step 2, both players immediately write this number down on their game boards.

Step 4: Players continue to alternate drawing cards until all six place-value positions for round 1 are filled. Players compare their numbers and read them aloud to each other.

The highest number created wins.

After several rounds, trainers may ask several questions:

- Is there anything you will do differently in the next round?
- What strategies have you discovered to be helpful in creating the largest number?
- What number were you hoping for each time a card was lifted?

Resources for Trainer

- Annex 3.

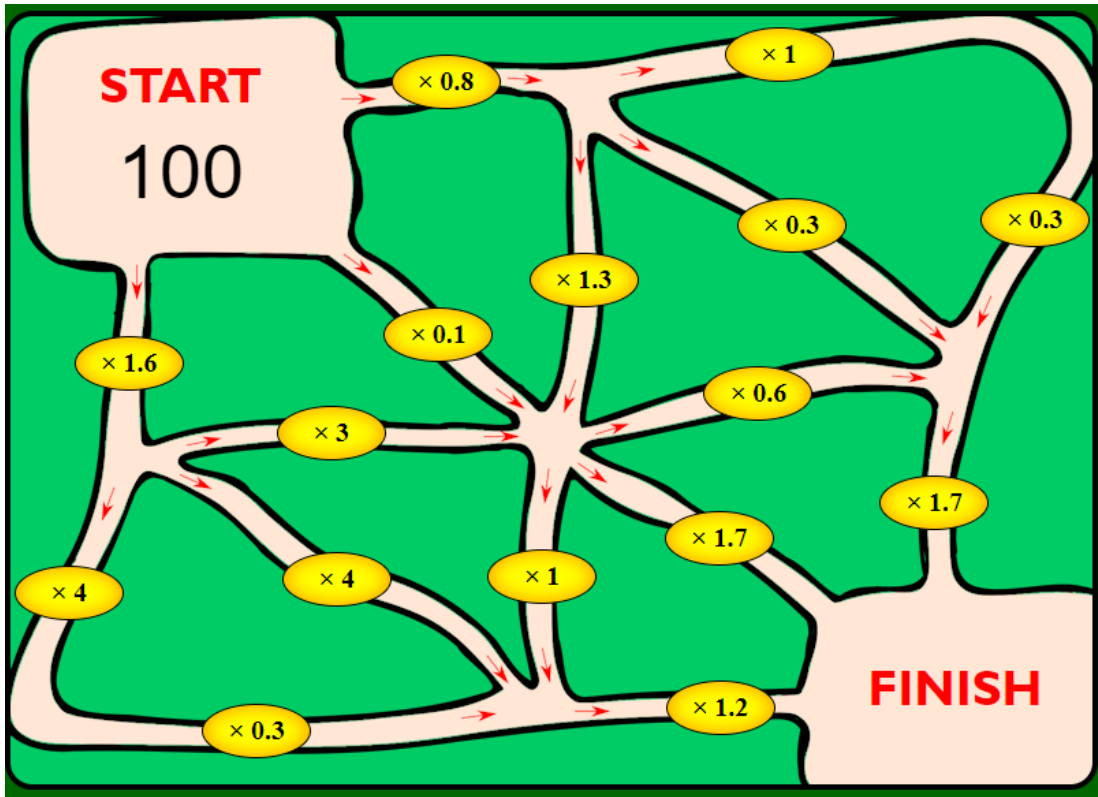
Other Resources needed

- Training Room.
- Pencils
- Deck of cards



ANNEXES

Annex 1



Annex 2

Hundreds	Tens	Ones	,	Tenths	Hundeths	Thousands



Annex 3

Round	1,000	100	10	1	0,1	0,01	Final Number
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							



LEVEL 3

Workshop Title: Family ties in fractions, decimals and percentages

CODE: L3.WA1.N3

LEVEL 3

WORK AREA 1: NUMBERS

PARTNER: FEH

Duration: 5 hours

Learning Outcomes Covered

Work Area 1: NUMBERS

Unit 1.11: Fractions

LO1: Define what a fraction is

LO2: Identify the numerator of a fraction

LO3: Identify the denominator of a fraction

LO4: Explain the features and the properties of the fractions

LO5: Determine the different meanings of fractions (sharing, division, ratio)

LO6: Define proper fractions (<1)

LO7: Recognize or define improper fractions (>1) which fractions are equivalent (e.g. $\frac{1}{2} = \frac{2}{4}$)

LO8: Recognize equivalent fractions

LO9: Transform one fraction in another equivalent

LO10: Have knowledge of the rules for adding and subtracting fractions

LO11: Have knowledge of the rules to solve additions and subtractions with fractions

LO12: Adding fractions with the same denominators

LO13: Summing fractions with different denominators

LO14: Subtracting fractions with the same denominators

LO15: Subtracting fractions with different denominators

LO16: Reduce a fraction to its minimum terms

LO17: Have knowledge of the multiplication and division rules of two fractions

LO18: Being able to multiply two fractions

LO19: Multiplies a natural number by a fraction



LO20: Being able to divide two fractions

LO21: Reduce a fraction to its simplest form (lowest terms)

LO22: Order and compare fractions

LO23: Order fractions with the same denominator

LO24: Order fractions where one denominator is a multiple of the other(s)

LO25: Order fractions with the same numerators

Unit 1.14: Conversions Percentages/Fractions/Decimals

LO1: Being able to convert a fraction to a percentage (and vice versa)

LO2: Express a fraction as a percentage

LO3: Express a percentage as a fraction

LO4: Being able to convert a decimal to a percentage (and vice versa)

LO5: Express a decimal as a percentage

LO6: Express a percentage as a decimal

Short introduction to the workshop

This workshop includes 5 activities. **Circles and lines.** These geometric forms are useful for a first activity related to fractions since it helps visualizing the basic components of a fraction. In this case, it helps identifying the basic components such as denominator and nominators. **Different ways to visualize a fraction.** With a set of squares and following a similar reasoning than the one indicated above geometric figures are the basis to understand fractions. The exercise is based on coloring and/or cutting different set of squares so that a visual effect takes place and is easier to understand the components of a fraction. Comparison between fractions also takes places in a more visual way. **Subtraction, addition, division and multiplication in fractions.** Several types of exercises where students may apply the knowledge of the different kind of operations that applies to fractions. Further explanations might be needed. It applies to fractions with same and different denominators and nominators. **Fraction War and Find Your Fraction Partner.** Games where that comparison between fractions takes places. It encourages students to understand and order different types of fraction in a competitive way so that they learn by playing. **Let's Play Basketball!** An easy way to understand statistics, fractions, percentages, decimals and their relation.

How the workshop (its activities) teach the LOs to the participants

This workshop aims to provide students with a clear understanding of what a fractions are (main components, rules of addition, subtraction, addition, minimum terms, and equivalents) by providing games, practical exercises and group activities. It also includes a mixed of activities in a quiz format. A brief explanation follows:

Circles and lines. These geometric forms are useful for a first activity related to fractions since it helps visualizing the basic components of a fraction. In this case, it helps identifying the basic components such as denominator and nominators.

Different ways to visualize a fraction. With a set of squares and following a similar reasoning than the one indicated above geometric figures are the basis to understand fractions. The exercise is based on coloring and/or cutting different set of squares so that a visual effect takes place and is easier to understand the components of a fraction. Comparison between fractions also takes places in a more visual way.

Subtraction, addition, division and multiplication in fractions. Several types of exercises where students may apply the knowledge of the different kind of operations that applies to fractions. Further explanations might be needed. It applies to fractions with same and different denominators and nominators.

Fraction War and Find Your Fraction Partner. Games where that comparison between fractions takes places. It encourages students to understand and order different types of fraction in a competitive way so that they learn by playing.

Let's Play Basketball! An easy way to understand statistics, fractions, percentages, decimals and their relation.

Challenges during the implementation of activities and how can be addressed

Lack of motivation. Unwillingness to play games or sports outdoors. Shyness of getting involved in contact games with others. Encouragement and willingness to make things easier from the trainer is an asset.



ACTIVITY 1: Components of a fraction (circles and lines)

This activity aims to provide the basic components of a fraction.

Step1: Provide **Annex 1** to students. They need to follow the instructions. Students have to indicate the parts of it as stated in the instructions.

Step 2: If any doubt, please check and remind the basic theory regarding numerators and denominators of a fraction.

Step 3: You can also provide **Annex 2**. This provides a different way of including different ways of naming fractions. Give it to students.

Step 4: If necessary, remind again about basic norms of fractions and its components.

You can check further information on the “*Resources for Trainer*” section.

Resources for Trainer

- Annexes 1 and 2
- **Explanation of denominator and nominator:**
<https://www.khanacademy.org/math/arithmetic/fraction-arithmetic/arith-review-fractions-intro/v/numerator-and-denominator-of-a-fraction>

Other Resources needed

- A Training Room
- Laptop and video projector.
- White board and markers.

ACTIVITY 2: Different ways to visualize a fraction

Step1: Give students **Annex 3**. Student must first choose the fraction and then cut the number of squares to have the equivalent in the table.

Step 2: Explain what a fraction is. Explain the differences between denominator and nominator. Check resources needed for further resources online.

Step 3: Give **Annex 4** now. There are several sets of figures. Student must say which one is equal, smaller or bigger than the ones drawn in table **Annex 3**.

Step 4: Explain how to compare fractions bigger, smaller and equal than others.

Step 5: Ask students to classify them into proper and improper fractions. Then explain the differences. You can check further explanations and examples on the videos available.

Resources for Trainer

- Annexes 3 and 4.
- Explanation of denominator and nominator:
<https://www.khanacademy.org/math/arithmetic/fraction-arithmetic/arith-review-fractions-intro/v/numerator-and-denominator-of-a-fraction>
- Explanations of comparison among fractions:
<https://www.khanacademy.org/math/arithmetic/fraction-arithmetic/arith-review-comparing-fractions/v/comparing-fractions>
- Explanation of proper and improper fractions:
<https://www.khanacademy.org/math/arithmetic/fraction-arithmetic/arith-review-mixed-number/v/comparing-improper-fractions-and-mixed-numbers>

Other Resources needed

- A Training Room
- Laptop and video projector.
- White board and markers.



ACTIVITY 3: Subtraction, addition, division and multiplication in fractions

This activity aims to provide a practical knowledge applicable to division, multiplication, subtraction and adding with fractions. There are different set of exercises in **Annex 5** and **Annex 6** for students to do.

Step1: Give each student **Annex 5**. Follow the instructions. Give 5 minutes to each group or students. Then compare answers. Make them discuss of the reasons why they are or not wrong.

Step 2: Explain or remind the rules of addition and subtraction if necessary.

Step 3: Get back to step 1. But now, provide to each student or pair of students with **Annex 6**. Follow the instructions. Give 5 minutes to each group or students. Then compare answers. Make them discuss of the reasons why they are or not wrong.

Step 4: Explain or remind the rules of addition and subtraction if necessary.

You can check further explanations and examples on the videos and links available.

Resources for Trainer

- Annexes 5 and 6.
- Answers for each of the exercise are in the same annexes.

Other Resources needed

- A Training Room
- Laptop and video projector.
- White board and markers.



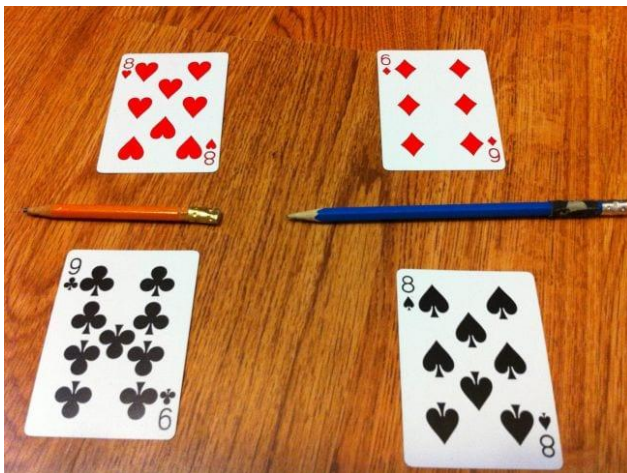
ACTIVITY 4: Fraction War

This activity tries to let understand and compare fractions as it was a cards game. You will need a deck of cards. You can also print the table added in the link of the section “Other Resources Needed”. **This may be helpful resources for students.**

Step 1: Split students in pairs. Give a deck of cards to each pair.

Step 2: Students deal two cards each, a numerator and denominator. They have to determine whose fraction is the largest. The winner keeps all four cards, and play continues until the cards are gone. The winner of the game is that one that keeps more cards. The quickest the more challenging and the more fun is the game.

Examples of how to deal both cards:



Step 3: You can repeat the game in pairs, one versus one or as many times as you want.

Resources for Trainer

- Printable table in this website. Others might be downloaded as well.
<https://s18670.pcdn.co/wp-content/uploads/2016/10/fractionwar.pdf?sfvrsn=0>

Other Resources needed

- A Training Room.
- Deck of cards.



ACTIVITY 5: Finds Your Fraction Partner

This activity aims to provide a practical exercise to get to know comparison between fractions. This activity is better to do it with a big class. You can find several cards in **Annex 7**.

Step1: Split the class in two teams. Give to each student a card. Students have to find their fraction partner, meaning someone that has a fraction number equivalent of their cards. They have 2 minutes.

Step 2: Once step 1 is finished. Each teams needs to put themselves in order from the smallest to the biggest equation. They have 5 minutes. They need to discuss why. Once finish, discuss it in front of the class.

Step 3: You can switch cards as many times as you want, following the same procedure than before. In the second part of the game, it also requires teamwork skills.

Resources for Trainer

- Annex 7
- You might want to add more cards. You can check on this website and create your own:
<https://www.helpingwithmath.com/printables/flashcards/fla0401fractions01.htm>

Other Resources needed

- A classroom
- More fraction cards if needed



ACTIVITY 6: Let's Play Basketball!

This activity aims at contributing to the knowledge of basic statistics with fractions, percentages and decimals and the relation among them based on basic activities. There are two possibilities for calculating statistics in this exercise. One, by playing a basketball match. Two, by playing a shooting competition. Use either based on your needs.

Step 1: Organize and play a small basketball match (1st option). Organize and play a shooting competition between two teams in a basketball court (2^o option).

Step 2: During the 1st option game, students must keep track of their shooting statistics (shots made over shots attempted). Ask them to remember their punctuation.

Step 3: Once the game finishes, ask students to convert each shooting fraction to a decimal and then a percentage. As a teacher, explain how fractions, percentages and decimals can be used to express the same quantity or numbers.

Step 4: During the 2nd option game, each player should keep track of the shooting fractions (shots made over shots attempted). Ask them to remember their punctuation and show it to the rest of the students.

Step 5: Each student must convert each shooting fraction to a decimal and then a percentage. Explain how fractions, percentages and decimals can be used to express the same quantity or numbers

Resources for Trainer

No resources needed for the workshop. This game might be adapted to several other games where statistics are available and there is use of fractions: football, bowling, foosball, tennis or others.

Other Resources needed

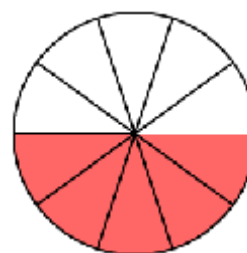
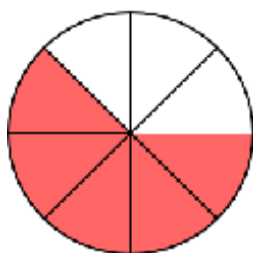
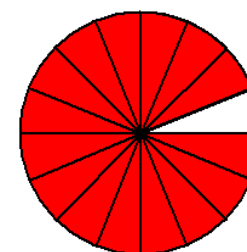
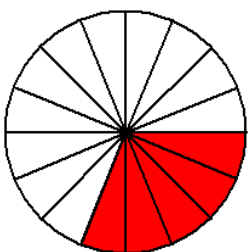
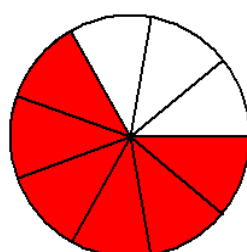
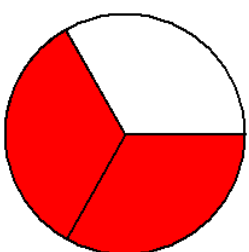
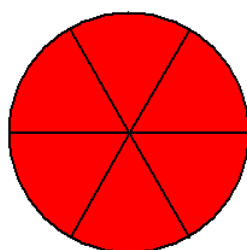
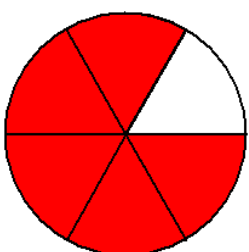
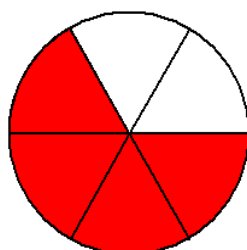
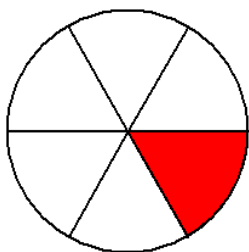
- Basketball court.
- Basketball ball.
- Basket.
- Paper ball.
- Class room.
- Laptop and video projector.
- White board and markers.

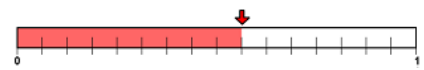
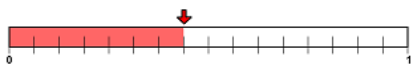
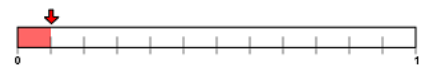
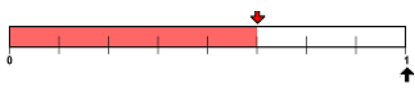
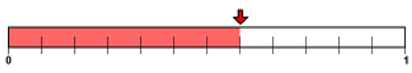
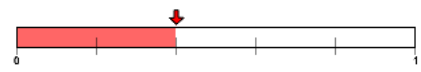
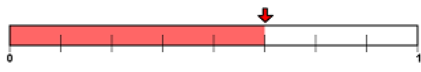


ANNEXES

Annex 1

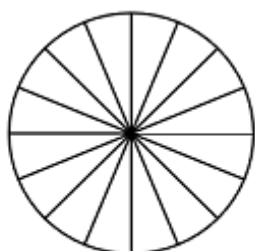
Write the numerator and denominator for the following:



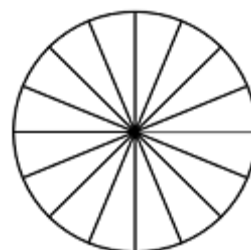


Annex 2

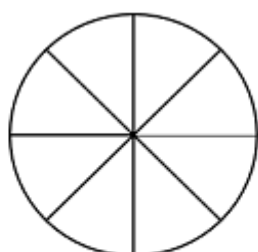
Shade the following as indicated:



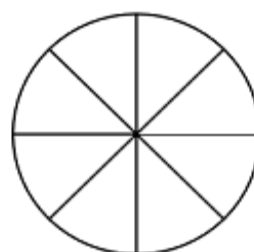
$\frac{5}{16}$ of the circle is shaded.



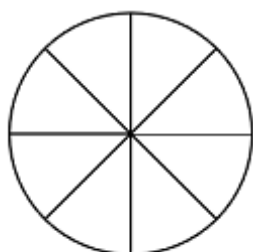
$\frac{15}{16}$ of the circle is shaded.



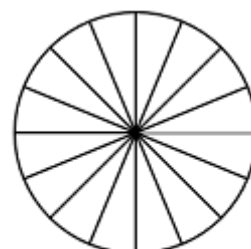
$\frac{7}{8}$ of the circle is shaded.



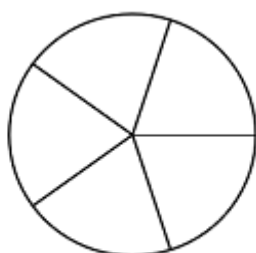
$\frac{1}{8}$ of the circle is shaded.



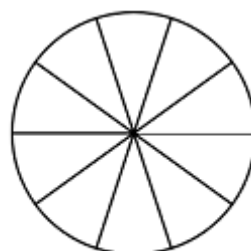
$\frac{5}{8}$ of the circle is shaded.



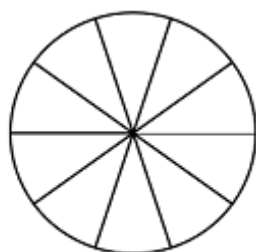
$\frac{10}{16}$ of the circle is shaded.



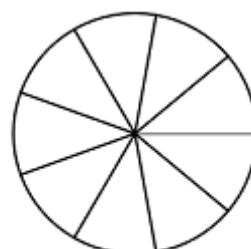
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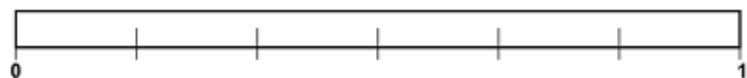
$\frac{3}{10}$ of the circle is shaded.



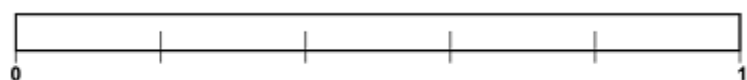
$\frac{7}{10}$ of the circle is shaded.



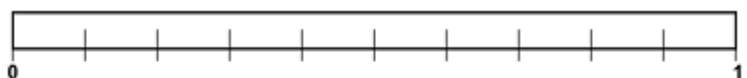
$\frac{7}{9}$ of the circle is shaded.



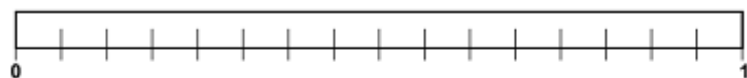
$\frac{5}{6}$ of the distance from 0 to 1 is shaded.



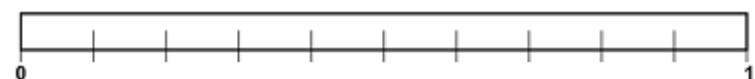
$\frac{4}{5}$ of the distance from 0 to 1 is shaded.



$\frac{4}{10}$ of the distance from 0 to 1 is shaded.



$\frac{3}{16}$ of the distance from 0 to 1 is shaded.



$\frac{7}{10}$ of the distance from 0 to 1 is shaded.

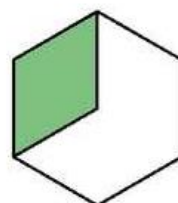
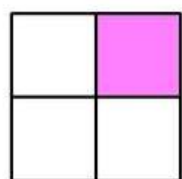
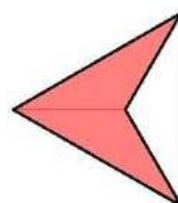
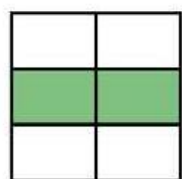
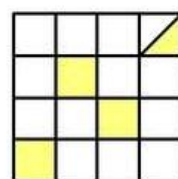
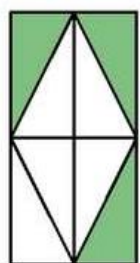
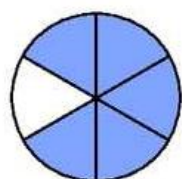
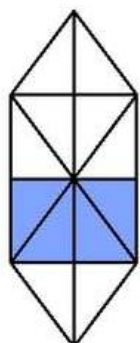
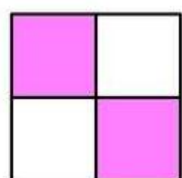
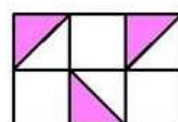
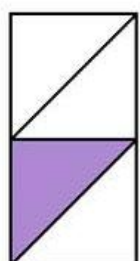
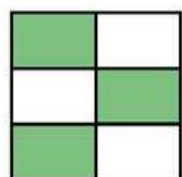
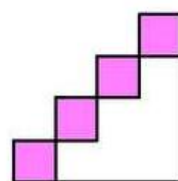
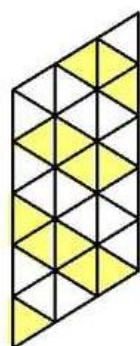
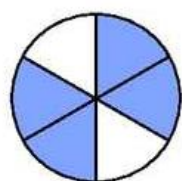


Annex 3

3/7	9/3	3/9	7/9
5/4	9/13	18/5	5/1
10/8	21/10	4/20	14/9
7/2	10/11	5/6	4/6

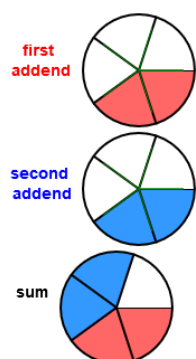


Annex 4

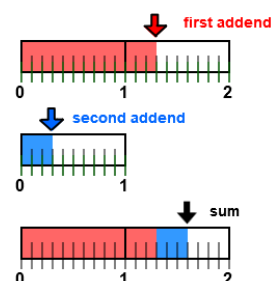
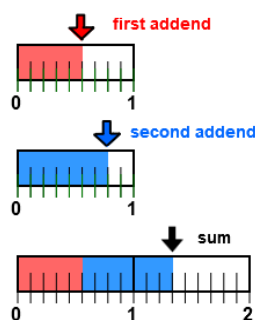
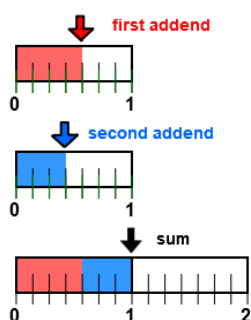
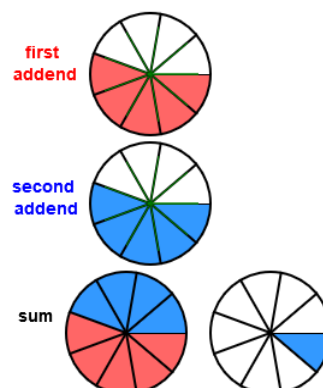
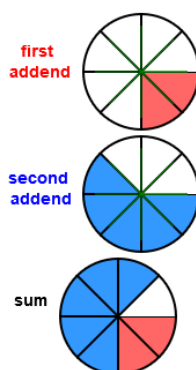
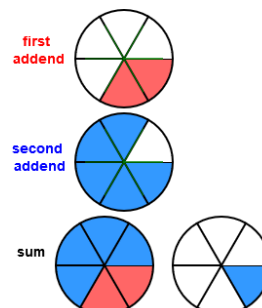
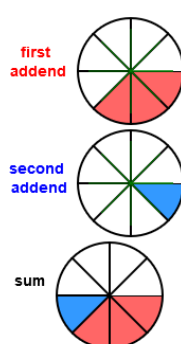


Annex 5

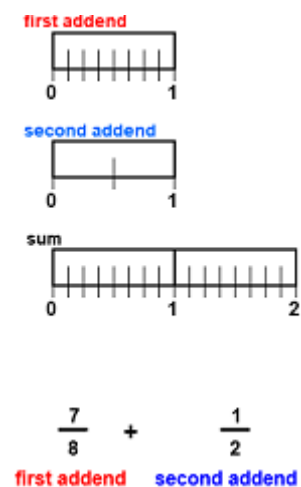
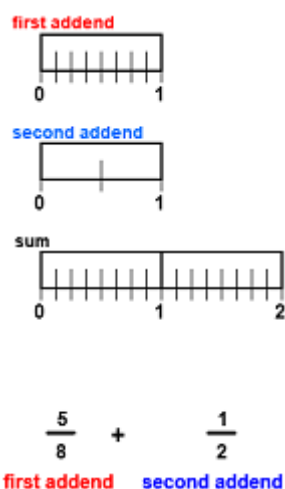
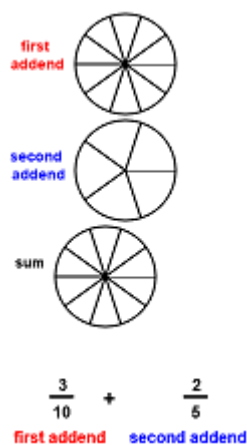
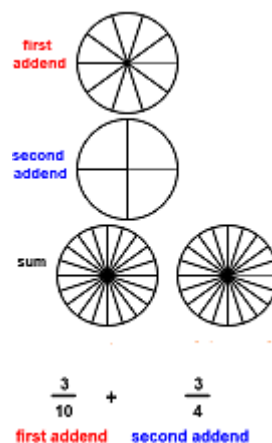
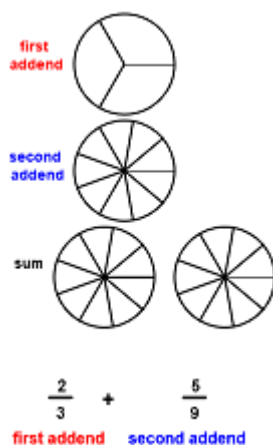
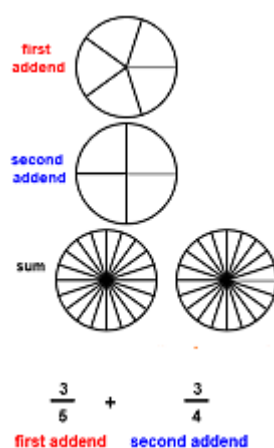
Write a number sentence that describes the picture (adding, subtraction)



$$\frac{2}{5} + \frac{2}{5} =$$

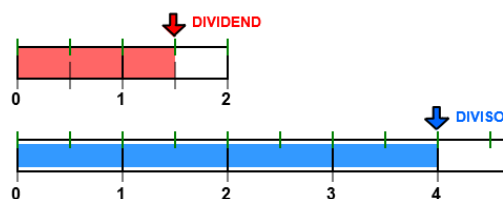
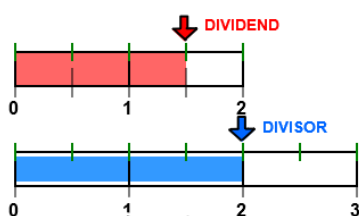
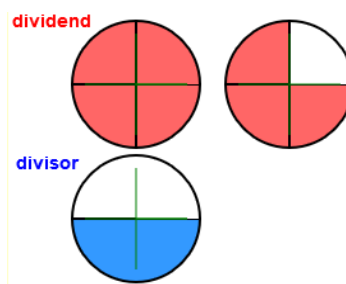
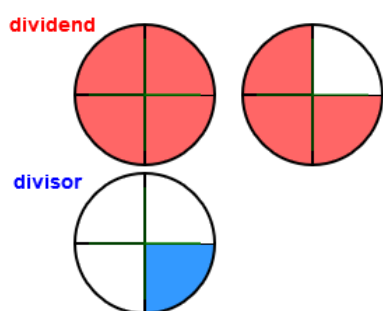
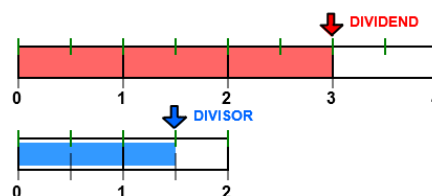
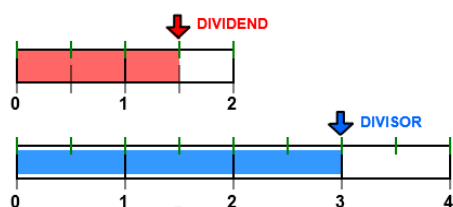
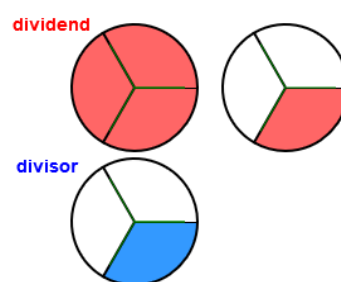
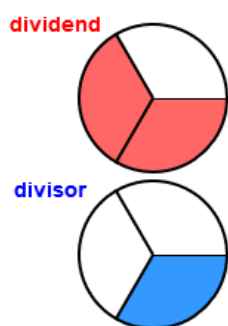


Color the picture to show the addends and sum and complete the number sentence that describes the picture.

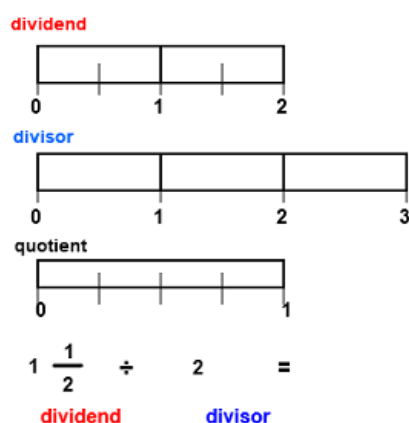
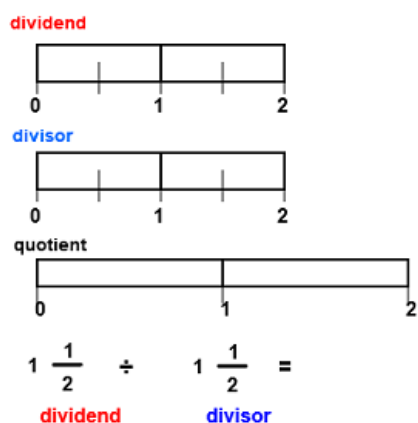
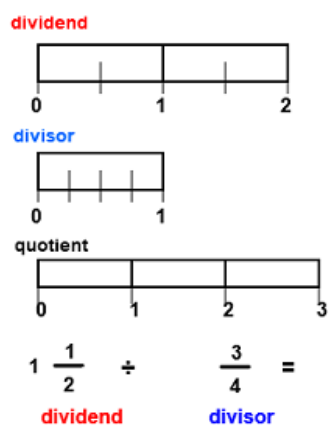
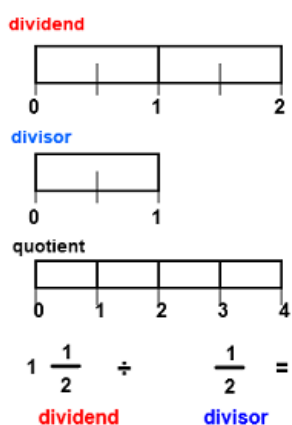
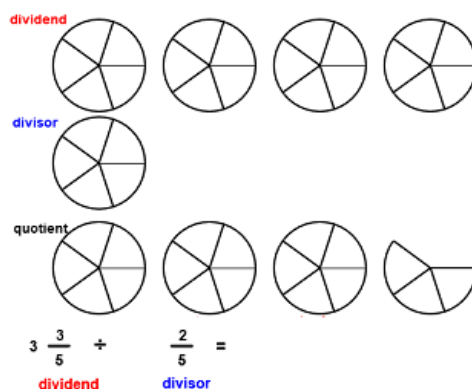
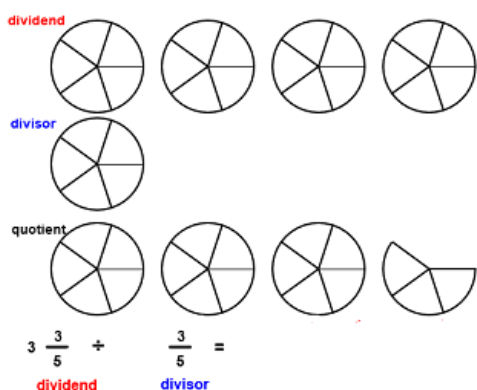


Annex 6

Write a number sentence that describes the picture.



Shade the indicated dividend and divisor. Then shade the quotient. Write a number sentence that describes the picture and simplify.



Annex 7

$\frac{1}{2}$	$\frac{2}{4}$	$\frac{5}{10}$	$\frac{10}{20}$
$\frac{1}{3}$	$\frac{2}{6}$	$\frac{3}{9}$	$\frac{10}{30}$
$\frac{1}{4}$	$\frac{2}{8}$	$\frac{3}{12}$	$\frac{10}{40}$

$\frac{2}{5}$	$\frac{4}{10}$	$\frac{5}{6}$	$\frac{2}{3}$	$\frac{2}{7}$
$\frac{1}{3}$	$\frac{3}{9}$	$\frac{4}{5}$	$\frac{12}{15}$	$\frac{1}{5}$



LEVEL 3

Workshop Title: Make easier your life with fractions	
CODE: L3.WA1.N4	
LEVEL 3	
WORK AREA 1: NUMBERS	
PARTNER: FEH	
Duration: 7 hours	
Learning Outcomes Covered	
WORK AREA 1: NUMBERS Unit 1.14 Conversion Percentages/Fractions/Decimals Being able to convert a fraction to a percentage (and vice versa). LO1: Express a fraction as a percentage LO2: Express a percentage as a fraction LO3: Express a decimal as a percentage LO4: Express a percentage as a decimal Unit 1.16 Solving simple problems with numbers LO1: Identify fractions related to everyday items (using objects or drawings) LO2: Represent a fraction in a variety of ways, based on a whole or a collection of objects LO3: Match a fraction to part of a whole (congruent or equivalent parts) or part of a group of objects, and vice versa LO4: Counting money LO5: Getting the change back LO6: Develop processes for mental computation LO7: Calculate a raise in salary LO8: Calculate the interest rates LO9: Calculate discounts LO10: Determine the operation(s) to perform in a given situation LO11: Translate a situation using a series of operations in accordance with the order of operations LO12: Identification and understanding of problems, hypothesis formulation and of solutions and their verification	



Short introduction to the workshop

This workshop wants to train all students in the agile calculation of percentages, fractions and decimals. So necessary in everyday life, to carry out activities, such as buying, calculating one's salary, or having good money management. That is why we focus on activities of changing fractions to percents and vice versa, as well as calculating percentages and decimals, which will enable our students to know how to read discounts on labels, and do the conversion.

How the workshop (its activities) teach the LOs to the participants

This workshop contains 4 didactic activities and three test questions that cover the previously broken down topics:

- FRACTIONS WITH DICE 2: Knowing how to go from fractions as parts of a whole to expression as a fraction, to decimal form or to percentages is an important task that must be consolidated. Reinforce the passage of fractions as part of a whole, as a decimal and as a percentage.
- HEXAGONAL PUZZLE OF FRACTIONS, DECIMALS AND PERCENTAGES: We present here 24 triangular tiles. Each triangle has a fraction, a decimal or a percentage on two or three of its sides. Reinforce the change from fraction to decimal or percentage and vice-versa.
- MANAGEMENT AND APPROXIMATION OF DECIMALS WITH CARDS: One of the most frequent errors, when comparing two decimal numbers, is the statement that 5,212 is greater than 5.3 as 212 is greater than 3. To work the order between decimals, and avoid this error, we propose a activity, with cards.
- THE MENTAL CALCULATION DECK OF DISCOUNTS: "Mental calculation together with the pencil and paper algorithms and the calculator is one of the necessary pillars on which to base any rigorous approach to numerical calculation."

In this chapter, the well-known group proposes a card game for the students to learn by playing to approximate the result obtained by applying a percentage, discount or percentage increase to a quantity.

Challenges during the implementation of activities and how can be addressed

Difficulty in understanding fractions.

Here you can find an interactive video of the passage of decimals, fractions and percents:

<https://www.youtube.com/watch?v=-Xt4UDk7Kzw>

In Khanacademy you can find a more detailed explanation:

<https://www.khanacademy.org/math/pre-algebra/pre-algebra-ratios-rates/pre-algebra-percent-decimal-conversions/a/converting-between-percents-fractions-decimals>



ACTIVITY 1: FRACTIONS WITH DICE 2

Game rules: Pair play

Step 1. Roll a die to find out who starts.

Step 2. Each player rolls the two dice and thus gets a fraction: the smallest result will be the numerator of the fraction and the largest will be the denominator.

Step 3 The player then places one of his pieces on one of the squares that correspond to his fraction. If all the corresponding squares are occupied, the player loses his turn. If the player makes a mistake placing his piece, he also loses his turn.

Step 4 The game is over after a preset time or when all squares are occupied.

Step5 The player with the most chips wins.

Resources for Trainer

- one board per pair
- 15 tiles for each player of different colors.
- 2 dice.

Other Resources needed

- Annex 1, printed,
- A Training Room
- A White Board and markers
- Laptop
- Video Projector



ACTIVITY 2: HEXAGONAL PUZLE OF FRACTIONS, DECIMALS AND PERCENTAGES

The game consists of joining the tiles together, joining the sides with two expressions, a fraction and its equivalent percentage, a decimal and its corresponding fraction or a percentage and its decimal. When you finish putting together the 24 pieces of the puzzle, the figure you get is a large hexagon like the one in the first image of this entry. This game is made with the help of the FORMULATOR TARSIA program.

Game rules: Individual or cooperative couples game.

Step 1 Each student or each pair should try to join the sides of the triangles by putting together a fraction and its equivalent percentage, a decimal and its corresponding fraction or a percentage and its decimal. In this way a large hexagon can be formed.

Step 2 Win the student or the pair that manage to form the big hexagon first.

Resources for Trainer

- 24 triangular cards per student or per pair of students.

Other Resources needed

- Annex 2, (printed, one per couple)
- A Training Room
- A White Board and markers
- Laptop
- Video Projector



ACTIVITY 3: MANAGEMENT AND APPROXIMATION OF DECIMALS WITH CARDS

Game rules:

The game consists of placing numbers in the tenths, hundredths and thousandths on a board in such a way that a number as close as possible to 1.5 is obtained.

Step 1 Game for two players. Each player has a board in front of him.

Step 2 On the table, place the ten cards with the numbers face down.

Step 3 Roll the dice to find out who will be in charge of the calculator and the game will begin.

Step 4 That player draws a card from the pile and places it in the tenths, hundredths or thousandths boxes.

Step 5 The other player draws another card and does the same.

Step 6 The game continues until the two players obtained a decimal number with three decimal places in their tables: 1,...

Step 7 Gain one point, the player who has gotten closer to 1.5. To find out, each player makes the difference between their number and 1.5. If there is any doubt, the player who has the calculator can proceed to clarify the situation.

For example, in the initial image it is seen that the player who has placed 1,497, gets a point in this game.

Resources for Trainer

Material needed for each couple:

- Some number cards from 0 to 9.
- Two game boards, one for each player. The units figure is already written on the boards.



- A dice.
- A calculator (teachers will decide which one to use or not)

Other Resources needed

- Annex 4 (printed,
- A Training Room
- A White Board and markers
- Laptop
- Video Projector



ACTIVITY 4: THE MENTAL CALCULATION DECK OF DISCOUNTS

We present here 4 of the 8 possible decks that the group proposes.

Game for four players.

Step 1 Following a turn, a team player takes a deck and the calculator. He shows a card and while the other three players on his team estimate the final price, he calculates it with the calculator.

Step 2 The player who comes closest to the result gets a point.

Step 3 When the 10 cards in the deck have been uncovered the round ends. The player who conducted the game in that round records his teammates' scores on the wall chart.

Step 4 The game ends when four rounds have been completed, clearly changing the player who leads in each round, with the four proposed decks.

Step 5 The player with the highest score wins.

Step 6 The four rounds can be performed over several days, taking up only one moment of the class.

Resources for Trainer

Material for each team:

- A deck of 10 cards.
- A calculator.
- A table of the results of all the teams in the group that is placed as a mural, such as the following:

The game is to estimate the price of an object after applying a certain increase or discount.

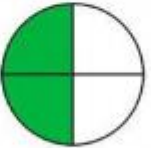
Other Resources needed

- Annex 4 (printed,
- A Training Room
- A White Board and markers
- Laptop
- Video Projector



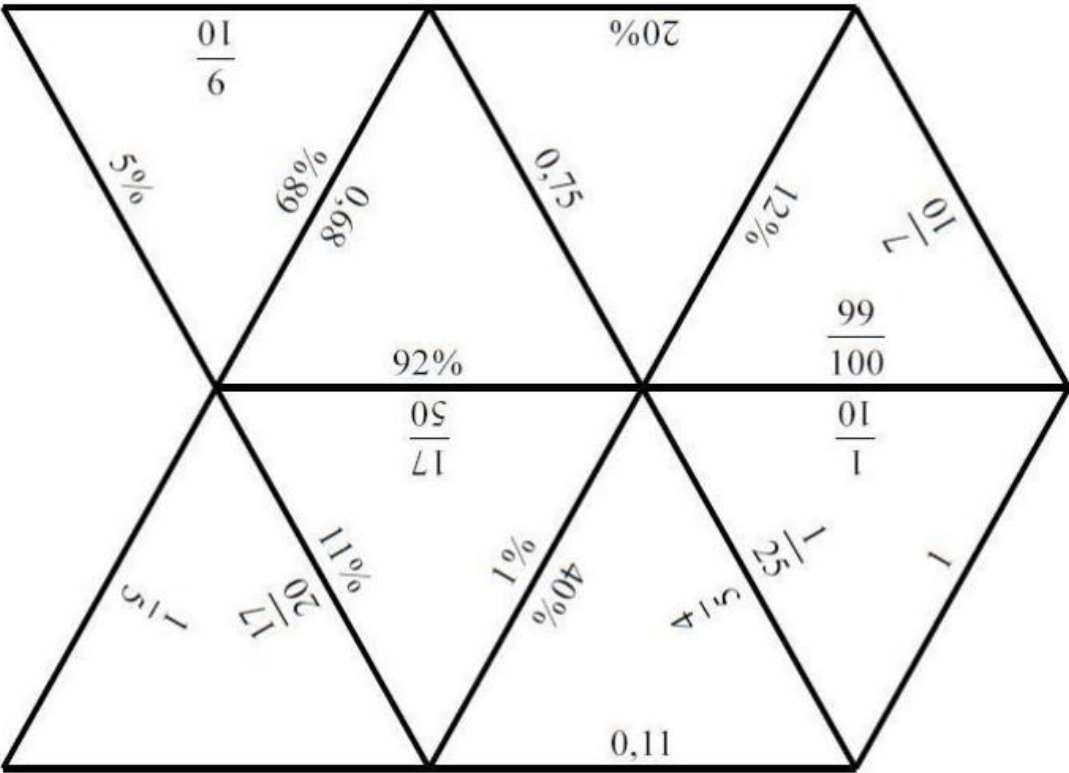
ANNEXES

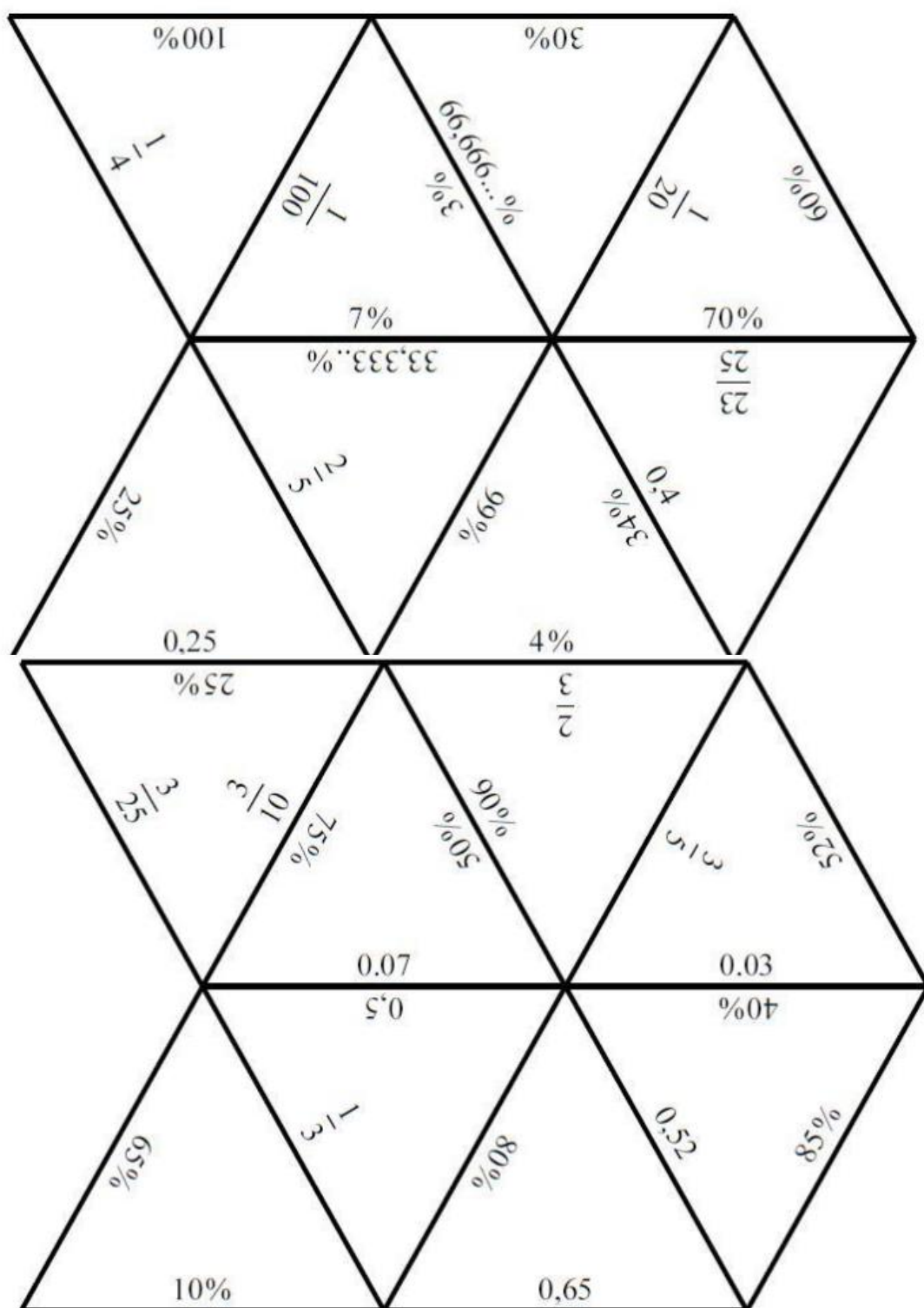
Annex 1

		25%		0,4
	100%		0,2	
	40%	0,6	50%	0,25
20%				
	75%			0,8
60%		0,5		



Annex 2





Annex 3

	0	
1	2	3
4	5	6
7	8	9

Player 1

1	.		
---	---	--	--

Player 2

1	.		
---	---	--	--



Annex 4

Precio= Price

De descuento= discount

Aumeno= Rise





WORK AREA 2: MEASUREMENT &
CONVERSIONS

LEVEL 3

Workshop Title: Metric Units of Measurement and conversions (Part 2)

CODE: L3.WA2.N5

LEVEL 3

WORK AREA 2: MEASUREMENT & CONVERSIONS

Partner: ERIFO

Duration: 5 hours

Learning Outcomes Covered:

WORK AREA 2: MEASUREMENT & CONVERSIONS

This WS address all the LO of the Unit 2.2 related to the conversion of units of measurement. This WS will provide knowledge and skills to reach the AC ladder 4. Learners will start from Units of weight, AC ladder 2; they will learn how to convert these units in order to transfer this skill to a practical daily activity (to cook and adapt recipes). The workshop will also allow to learn how to make conversions with units of length, area, volume, time.

2.1 Metric Units of Measurement

LO1: understand and list units of weight (used in the local, national context) (t, kg, gr, g)

2.2 Converting numbers from one unit to another

LO1: Convert between standard units of **length**.

LO2: Convert between standard units of **area**.

LO3: Convert between standard units of **volume**.

LO4: Convert between standard units of **weight**.

LO5: Convert between standard units of **time**.

2.5 Solving simple problems with measurements and conversions

LO: Adapt a recipe which is calculated for 4 persons, to 3 (6) persons. (Plus and Minus)



Short introduction to the workshop

The Workshop is structured into 4 activities in order to provide specific knowledge and help low skilled adults in developing skills to move from level 2 to level 4 of the AC ladder, for what concern the themes addressed in WA 2. Topics addressed:

In the first activity, learners will get to know how to measure weight and express it through the use of units of weight.

In the second activity, they will learn how to make conversions between standard units of weight (kg into g..)

In the third activity, learners will be involved in different exercises in order to learn how to make conversions between other standard units (units of length, area, volume, time)

The last activity will be more practical and will push learners to apply what they have learned in a real-life context. They will be asked to adapt a recipe to a different number of guests).

How the workshop (its activities) teach the LOs to the participants

The workshop is based on an experiential approach therefore activities refer to real-life situations and examples of daily life tasks. This approach is particularly effective as students are asked to deal with challenges they may have already encountered in their daily life and thus they will become quickly aware of the importance of numeracy skills and be more motivated to learn. Experiential learning focuses on learners reflecting on their experience of doing something, so as to gain conceptual insight as well as practical expertise. This approach will be fundamental to motivate learners and ensure the achievement of the final objective of the workshops: the development of the knowledge and skills needed in today's society, especially in the world of work.

Challenges during the implementation of activities and how can be addressed

This Workshop foresees the active involvement of participants.

The diversification of the activities will guarantee major possibility to gain the learners attention and interest. Moreover, the WS is developed to permit them to find easily connections with situations that they tackle every day.

The cooking time should be an immersive and amusing activity.



ACTIVITY 1: What's the weight?

Step 1: Brief review on the metric measures of weight (milligrams, centigrams, grams, kilograms, etc.) and their symbols (mg, cg, g, kg, etc) using illustrated cards (see example in **Annex 1**). On each card there will be a picture portraying any object/material and the most frequently used unit to measure the weight of that material/object. For example, you will have a card with flour and kg as unit of weight, a big box of fruit and q as unit of weight.

Step 2: using a scale, learners have to weigh different objects they find in the classroom and outside and write down their weight on a sheet

Step 3: they have to indicate the weight using different weight's unit (e.g. the pen weighs ... grams; the pinecone weighs ... centigrams).

Step 4: the mentor asks learners to compare weights and write down all the objects whose weight has been measured in descending order of weight

Resources for Trainer

- Illustrated cards which can be prepared by the trainers following the example provided in Annex 1 (weight, but it is possible to use other units, like length or area or volume ones)
- Scale

Other Resources needed

- Training room
- Outdoor space



ACTIVITY 2: 1 kg is how many g?

Step 1: the mentor will guide students to develop conversion skills through an empirical approach; he/she will show learners the scale and ask them to check how many g there are in one Kg; learners will realize that $1 \text{ Kg} = 1000\text{g}$, therefore by multiplying 1 Kg by 1000 they can convert the unit from Kg into g.

Step 2: after this introduction learners will be ready to study the mechanism for equivalences and convert units of weight

Step 3: learners have to understand how to convert from one weight's unit to another one. Thus, they have to draw on a paper a chart which list the weight unit' symbols from the heaviest to the lightest:

t	q	Mg	kg	hg	dag	g	dg	cg	mg

Step 4: The exercise consists in writing a quantity of weight (e.g. 1 g) in the chart, and converting it into another weight's unit.

Step 5: it is possible to do examples with daily situation that learners face with. For example, with 1 kg of bread, how many sandwiches you can prepare, considering that each one should weigh 100 g?

Resources for Trainer

- Equivalence table
- Smart board or blackboard to write examples

Other Resources needed

- Working sheets for learners



ACTIVITY 3: How tall are you?

Step 1: the mentor will guide students to develop conversion skills through an empirical approach; he/she will show learners the scale and ask them to check how many cm there are in one m; learners will realize that $1\text{ m} = 100\text{ cm}$ therefore by multiplying 1 m by 100 they can convert the unit from m into cm.

Step 2: after this introduction learners will be ready to study the mechanism for equivalences and convert units of length

Step 3: learners have to understand how to convert from one length's unit to another one. Thus, they have to draw on a paper a chart which list the weight unit's symbols from the heaviest to the lightest:

Km	hm	dam	m	dm	cm	mm

Step 4: The exercise consists in writing a quantity of length (e.g. 1 g) in the chart, and converting it into another length's unit.

Step 5: it is possible to do examples with daily situation that learners face with. For example, if the distance between my house and the school is 23 Km, how many m will it be?

Resources for Trainer

- Equivalence scale

Other Resources needed

- Training room
- Working sheets for learners



ACTIVITY 4: Cooking Numbers

Step 1: all the group, both mentor and learners, have to discuss about a regional recipe they usually cook in this precise period of the year (perhaps a public holiday is approaching and there is a typical dish that is prepared on this occasion)

Step 2: learners have to define the exact recipe for 4 people, while indicating the quantities.

Step 3: the learners have to convert from the chosen units of weight (e.g. kilograms) to another one (e.g. grams)

Step 4: they have to adapt the chosen recipe, which was originally calculated for 4 persons, for half the people (2 people)

Step 5: if there is the possibility to use a Kitchen, they can cook one or more recipes using the method they have learnt

Resources for Trainer

- None

Other Resources needed

- Training room
- Outside
- Kitchen



ANNEXES

Annex 1



LEVEL 3

WORK AREA 3: SHAPES AND SPACE

Workshop Title: “Exploring Maths within a world of 2D Shapes”
CODE: L3.WA3.N6
LEVEL:3
WORK AREA 3: SHAPES AND SPACE
PARTER: DIMITRA
Duration: 5-7 hours
Learning Outcomes Covered:
Level 3 • Identify the appropriate formulas to calculate area, perimeter of 2D Shapes • Calculate the area, perimeter of 2D shapes by using the appropriate formulas
Short introduction to the workshop
The workshop focuses on describing and implementing the basic geometry calculations. The activities include: short presentation of basic geometric calculations., provision of formulas and guidance on how to use 3d geometric calculations, practical exercise to fully understand and implement the presented formulas.
How the workshop (its activities) teach the LOs to the participants
The workshop for basic geometric calculations not only provides a substantial theoretical framework for the trainees but also presents a series of practical activities connected to it. In that way, the trainee will have the opportunity to combine his/her theoretical knowledge from the first activities with practical exercises, putting his/her just obtained knowledge in action.



Challenges during the implementation of activities and how can be addressed

This workshop could be quite challenging for people that have a low scaled mathematical thinking ability. In order to moderate this problem, the workshop includes a step by step educational procedure that begins with easier calculations and continues with more advanced so as to be easy for the trainee.



Step by step instructions for the trainers/mentors

ACTIVITY 1: 2D shapes in action

STEP 1: Make a short presentation of basic geometric calculations.

STEP 2: Provide an exercise sheet with a formula and ask them to design the correct object (at least three formulas)

STEP 3: Then ask them to choose the surface of different objects in the classroom (a book, a desk, a post-it note) and in combination with those they designed to calculate the area and perimeter. Ask them to note them down and then present the results to the classroom.

Resources for Trainer

- A presentation with 2D shapes and basic calculations of area, diameter and area
- Area of a Rectangle, Triangle, Circle & Sector, Trapezoid, Square, Parallelogram, Rhombus, Geometry Provision of exercise sheet.
<https://www.youtube.com/watch?v=JnLDmw3bbuw>
- Slideshare Net, 2D and 3D Geometry Formulas eBook
<https://www.slideshare.net/PDF-eBooks-For-Free/geometry-formulas-2d-and-3d-ebook> (accessed on 20/9/2019)

Other Resources needed:

None



WORK AREA 4: HANDLING
INFORMATION

LEVEL 3

Workshop Title: Highly Unlikely, or is it? Visualizing Data and Solving Simple Problems Using Information Handling Skills

CODE: L3.WA4.N7

LEVEL:3

WORK AREA 4: HANDLING INFORMATION

PARTER: METROPOLIS

Duration: 3h

Learning Outcomes Covered:

Work Area 4: HANDLING INFORMATION

Unit 4.1. Visualizing Data

LO1: Identify the common types of charts:

- Column/Bar graphs
- Pie charts
- Line graphs.

LO2: Have knowledge of the main charts and graphs commonly used to visualize data

Unit 4.4 Solving Simple Problems Using Information Handling Skills

LO1: In simple situations, identify the events that are most likely to happen

LO2: Describe and compare facts and events

LO3: Develop forecasts and hypotheses

LO4: Know the different probabilities behind real life situations

Short introduction to the workshop

The “handling information” work area focuses on the visualization of information, specifically by understanding charts and diagrams.

This workshop deals with data visualizations found in our daily lives, such as when presenting with election results or when opinion surveys are published.

Proportions such as those of the dietary pyramid or of population growth can emphasize why the visualization of numbers and data is so helpful.

After looking at a variety of examples, different types of visualization forms and



their names are summarized. The vital information needed to correctly interpret a diagram; a bar chart, etc is also reviewed.

Another focus of this workshop is on solving simple problems with regards to probabilities. The learner will first receive as well as gather their own examples of situations/problems demanding knowledge to assess the probability of a certain event or outcome to happen. Those situations, e.g. the safety of different contraceptives, winning the lottery, weather forecast, unintended side effects of meds etc. will be analyzed step by step by using the rules of probability calculation.

How the workshop (its activities) teach the LOs to the participants

This workshop aims to provide students with a clear understanding of the visualization of data, percentages, probabilities and forecasts. This is accomplished through numerous methods:

First, diagrams are explained, and different formats are evaluated. Next, the view is broadened, and we discuss the use of diagrams to display progressions. Predictions such as election forecasts are scanned with actual results. It is always checked which types of charts are used for presentation.

Challenges during the implementation of activities and how they can be addressed

The availability of technical equipment is relevant here for many activities.

Although an effort to provide alternatives is always made, learning via digital media and tools is preferred in order to practice and implement the digital competences that are already part of the numeracy workshops.

However, if the current digital competences of the learners are not sufficient to apply the proposed methods, tasks can be extended from small group work to the whole group of learners. Then, the teacher would use the technology and explain to the learners how to proceed. A smart board or a laptop with a beamer would be very helpful in this case.



ACTIVITY 1: Visualization of numbers and data

Step 1: To make clear why the visualization of numbers and data is so helpful, the teacher shows the dietary pyramid and population growth graphics. Using these graphics, they explain what is seen and what information may be derived from it.

Step 2: The teacher shows pictures of common chart types: column/bar graphs, pie charts, line graphs. As a group, they talk about which type of diagrams they have seen before and in what context that was.

Step 3: The teacher explains for each of the 3 types of election results diagrams in the appendix which party received the most votes and how this can be seen. The teacher also explains how the line chart shows how the results have changed over time in different elections.

Step 4: The students are asked to find out in the group which party received the fewest votes and explain how this can be seen.

Step 5: The group watches a video (see resources – video on pie chart) together in order to view a more complex pie chart. Afterwards the teacher asks if there are any further questions. Watch another video on reading line graphs (see resources – video on line graphs).

Resources for Trainer

- See Pictures of different charts in [Annex 1](#)
- Video on complex pie chart: <https://www.khanacademy.org/math/pre-algebra/pre-algebra-math-reasoning/pre-algebra-picture-bar-graphs/v/reading-pie-graphs-circle-graphs>
- Video reading line graphs: <https://www.khanacademy.org/math/statistics-probability/displaying-describing-data/more-on-data-displays/v/u08-l1-t2-we2-reading-line-graphs>

Other Resources needed

- Downloaded videos or internet access
- Large printout of the 3 diagrams or alternatively Smartboard or copies for all students



ACTIVITY 2: Charts in daily life

Step 1: The learners will separate into small teams (3-4 persons) and have the task to go to a) the newsstand and pick a newspaper and/or b) newspaper online (each group a different one!) and go through the sections.

Step 2: For each section where they find a chart, they take a picture (or screenshot) of it and take notes from what section it was and what it is supposed to illustrate.

Step 3: The groups collect all graphs and produce a small report to present to the other groups (what type of chart is it, what does it indicate according to the texts, what section is it from.)

Resources for Trainer

No special resources required, but the trainer should think of enough websites in case the groups all end up with the same websites

Other Resources needed

- Sufficient equipment with which small groups can search the Internet for newspapers and diagrams contained therein.
Alternatively, time and close distance to a newsstand where you can search for diagrams in newspapers and take pictures of them with a camera.



ACTIVITY 3: Understanding graphics in elections and forecasts

Step 1: The teacher explains the terms forecasts and hypotheses.

Step 2: Learners watch a video of the last election forecast (e.g. presented in the news) and another video of the election results in their country.

Step 3: In small groups, the learners analyze step by step what pictures (graphs, diagrams) they see and how they are explained by the moderator in the video (what words are used).

Step 4: The groups discuss what they understand, gather questions and collect what they find most interesting about it.

Step 5: After the small group work, every group shows their results and questions.

Step 6: The learners should describe why a specific type of chart seems to be useful to use to illustrate data.

Resources for Trainer

- Country-specific videos on election forecasts and videos on the numerical evaluation of election results;
For example: <https://www.youtube.com/watch?v=LmXqtH7UbrY>

Other Resources needed

- Laptops, beamer or Smartboard to play the videos. Multiple laptops or tablets would be helpful as learners can then work in small groups and pause and repeat the video as they need to.



ACTIVITY 4: The probability in forecasts

Step 1: Observe a 3-day (or longer) weather forecast (through internet or newspaper articles), that includes the information of how probable a certain weather condition will happen.

Step 2: Categorize the information using a simple prognosis like “quite probable”, “about fifty-fifty”, and “not very probable”.

Step 3: The learners should compare the current weather with the previous forecasts of the last days. The learners report to each other how the results have changed. The group will discuss how the accuracy of forecasts differs.

Step 4: The forecasts of election results already observed in previous activities are relatively precise, while the weather forecasts are often not correct. In terms of probability, let it now be seen what coincidence means. Now the comparison is to be drawn to chance. Ideally, the learners should play an online wheel of fortune alone in small groups. This is a simple online spinner which can easily simulate independent events. Learners can set the number of spins, and there's a table which automatically tracks and records the events.

Step 5: Another exercise in distinguishing Certain, probable, unlikely, and impossible. The learners should play the online game independently.

Resources for Trainer

- Screenshots from weather forecasts of the last 3 days, alternatively: experts from weather forecasts from newspapers.
- Adjustable Online Spinner:
<https://illuminations.nctm.org/tools/mathresources/NCTMAdjustableSpinner.html>
- Online game - Certain, probable, unlikely, and impossible:
<https://www.ixl.com/math/grade-3/certain-probable-unlikely-and-impossible>

Other Resources

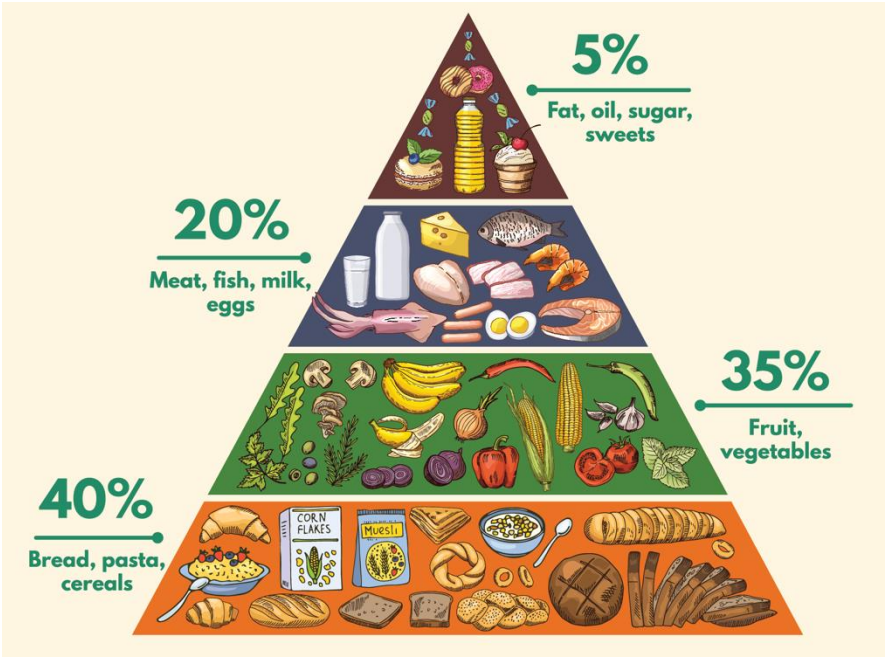
none



ANNEXES

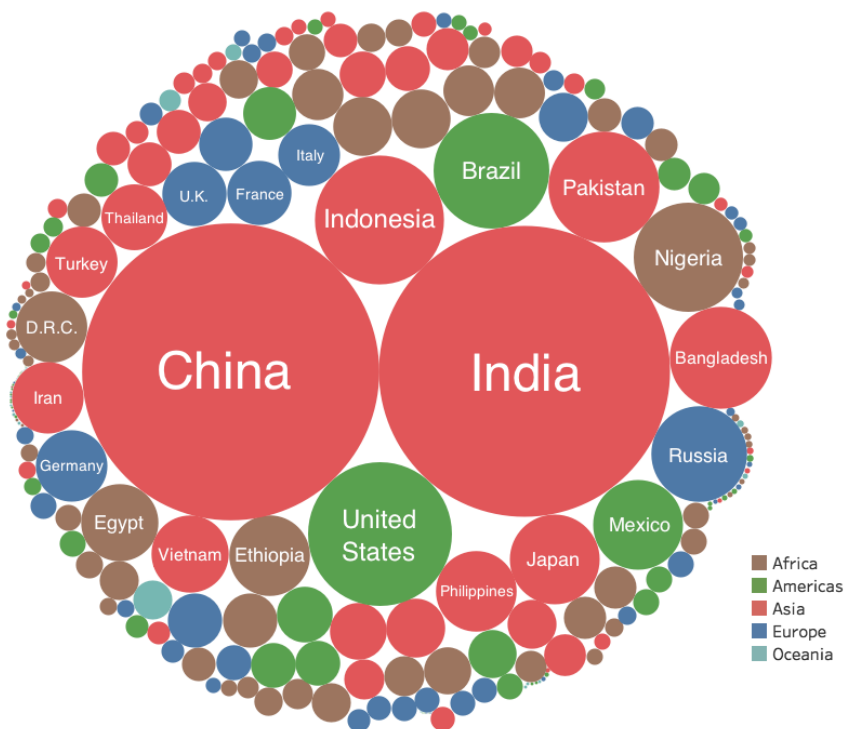
Annex 1

Dietary pyramid:



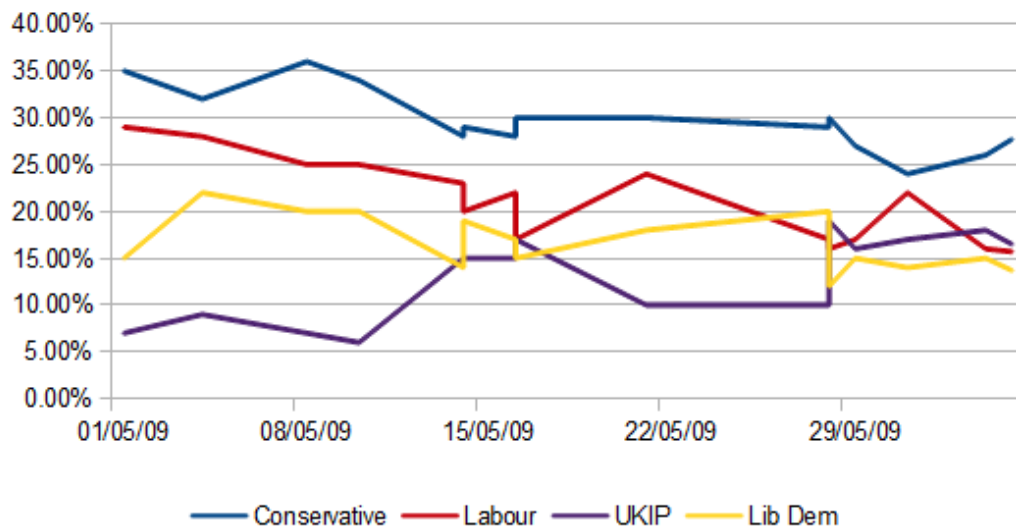
Population size:

Countries by Population Size

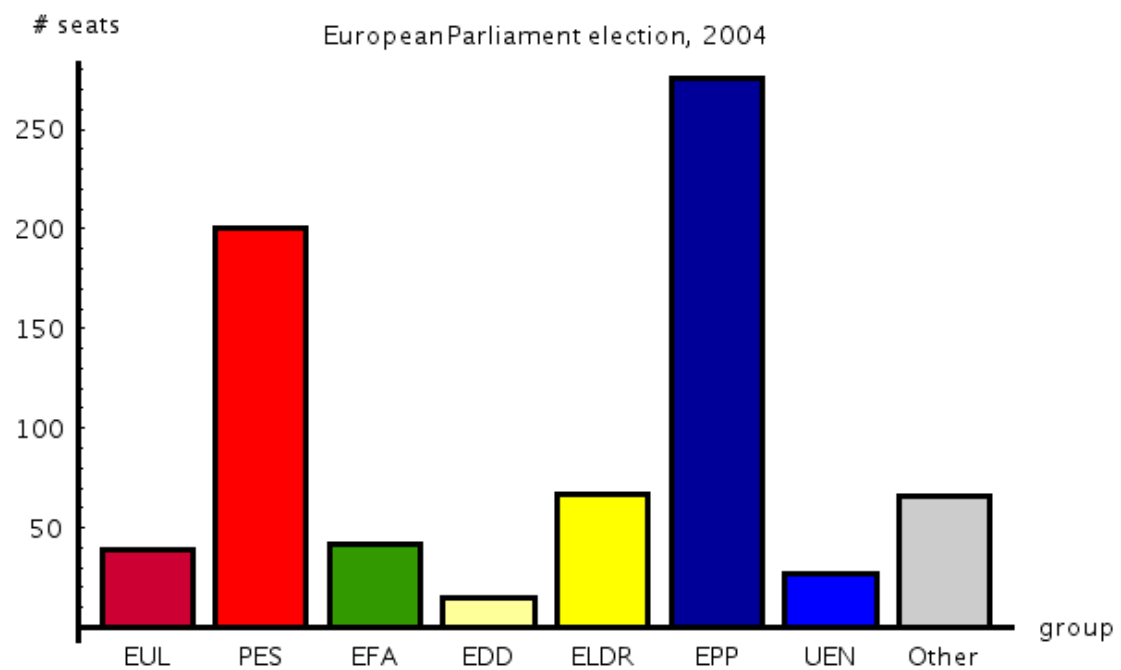


Line chart:

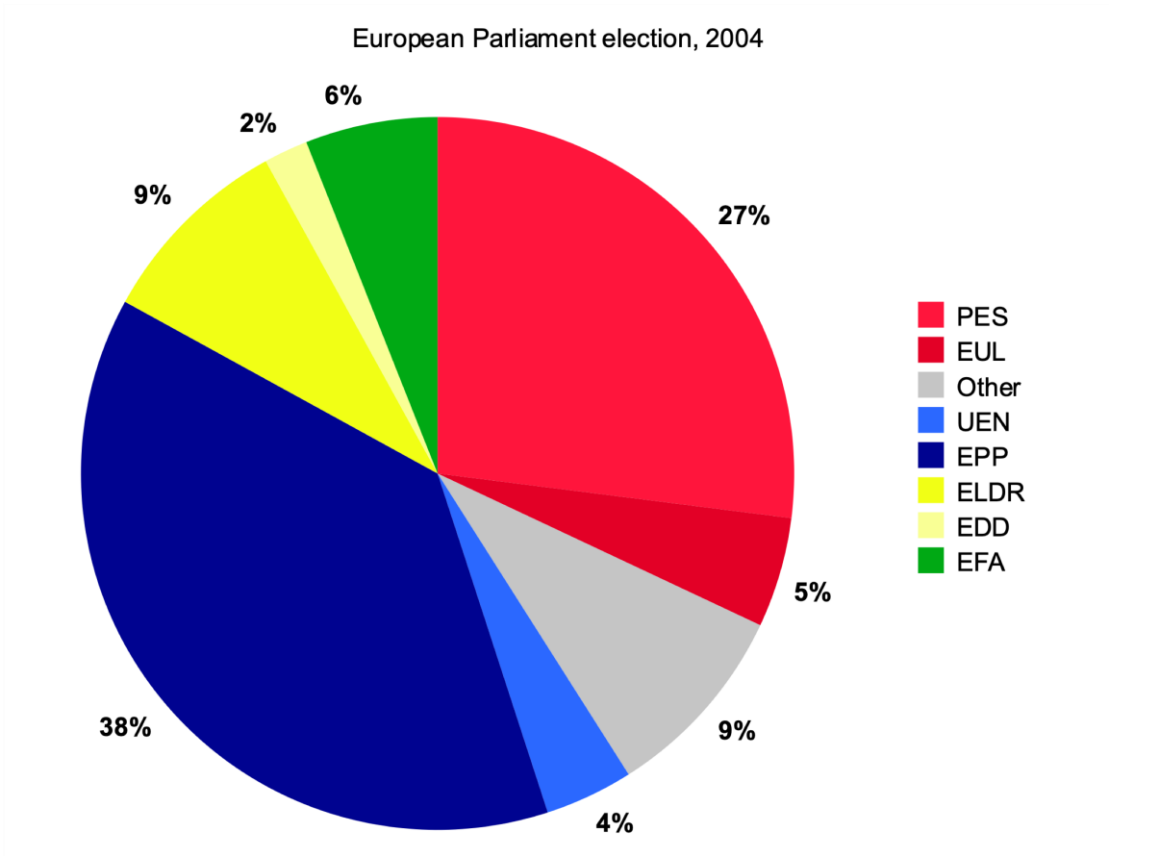
Opinion Polling for the European Parliament election, 2009 (United Kingdom)



Column/Bar graphs:



Pie Chart:



168



Active Citizenship Level 4

WORK AREA 1: NUMBERS

LEVEL 4

Workshop Title: Working with Percentages and performing mental calculations
CODE: L4.WA1.N1
LEVEL 4
WORK AREA 1: NUMBERS
PARTNER: EDITC
Duration: 7 hours
Learning Outcomes Covered:
Work Area: NUMBERS Unit 1.15 Solving Problems with Percentages, Decimals and Fractions LO1: Compare two or more percentages (10% of 100 and 30% of 200) Unit 1.16 Solving simple problems with numbers LO1: Match a fraction to part of a whole (congruent or equivalent parts) or part of a group of objects, and vice versa LO2: Develop processes for mental computation LO3: Calculate a raise in salary LO4: Calculate the interest rates LO5: Calculate discounts
Short introduction to the workshop
The workshop includes 4 activities. The first activity is a game that aims to provide a good understanding what a fraction, a percentage and a decimal is and how these three are linked. This is an activity that each participant needs to perform on his/her own. The 2nd activity is a game and focuses on making mental calculations with speed. The 3rd activity is a group activity and it is drawn from real life situation where participants need to calculate how much fat is contained in various food products. The last activity is a game, the well know “weak link” where students in groups need to provide answers to a series of problems that revolve around percentages.



How the workshop (its activities) teach the LOs to the participants

This workshop aims to provide students with a clear understanding on percentages, fractions and decimals and how they relate. There are numerous ways to teach fractions:

- **Part- Whole.** Using the part-whole construct is an effective starting point for building meaning of fractions (Cramer & Whitney, 2010). Part-whole can be shading a region, part of a group of people (3 out of 5 of the class went on the field trip), or part of a length (we walked 31 2 miles).
- **Measurement** involves identifying a length and then using that length as a measurement piece to determine the length of an object. For example, in the fraction $\frac{5}{8}$, you can use the unit fraction $\frac{1}{8}$ as the selected length and then count or measure to show that it takes five of those to reach $\frac{5}{8}$. This concept focuses on how much rather than how many parts, which is the case in part-whole situations (Behr, Lesh, Post, & Silver, 1983; Martinie, 2007).
- **Division.** Consider the idea of sharing €10 with 4 people. This is not a part-whole scenario, but it still means that each person will receive one-fourth ($\frac{1}{4}$) of the money, or 2 2 euros.. Division is often not connected to fractions, which is unfortunate. Students should understand and feel comfortable with the example here written as $\frac{10}{4}$, $10 \div 4$, $2 \frac{2}{4}$, and $2 \frac{1}{2}$ (Flores, Samson, & Yanik, 2006).
- **Operator.** Fractions can be used to indicate an operation, as in $\frac{4}{5}$ of 20 square feet or $\frac{2}{3}$ of the audience was holding banners. These situations indicate a fraction of a whole number, and students may be able to use mental math to determine the answer. This construct is not emphasized enough in school curricula (Usiskin, 2007). Just knowing how to represent fractions doesn't mean students will know how to operate with fractions, which occurs in various other areas in mathematics (Johanning, 2008).
- **Ratio.** The concept of ratio is yet another context in which fractions are used. For example, the fraction $\frac{1}{4}$ can mean that the probability of an event is one in four. Ratios can be part-part or part-whole. For example, the ratio $\frac{3}{4}$ could be the ratio of those wearing jackets (part) to those not wearing jackets (part), or it could be part-whole, meaning those wearing jackets (part) to those in the class (whole).

The activities described below focus on making calculations with percentages, comparing percentages of a number, understand the relation between percentages, decimals and fractions and finally they provide the insides on how to perform mental calculations with percentages

Challenges during the implementation of activities and how can be addressed

Reluctant to participate in the game out of fear: In the activities that we may have a higher degree of difficulty we group the participants so the feeling of inadequacy is decreased



ACTIVITY 1: Percentage Basics and relation with fractions and decimals

This activity aims to provide a thorough understanding of percentages and how to compare them

Step1: Provide each student with a drawing of a square which is divided to a 10X10 smaller squares (see Annex 1)

Step 2: Ask them to shade 10 small squares

Step 3: Ask them to indicate the shaded part of the square as a percentage: Result is 10%. Explain what % means. Per cent (cent in latin means hundred) Therefore 10% means 10 out of a hundred

Step 4: Ask the students to shade 10 more small squares. Ask them to indicate what percentage of the square is now shaded.

Step 5: Provide the students with 2 squares (see Annex 2) and ask them to shade 110%.

Step 6: Provide the students with a square which is split in 10 equal parts and 7 of the parts are shaded (see Annex 3). Ask the participants to express this as a

- a) Fraction
- b) Percentage
- c) Decimal

Step 7: Explain how fractions, percentages and decimals can be used to express the same portion of a whole.

S. Watch the video!



Resources for Trainer

- Annexes 1, 2 and 3
- Video: <https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-ratios-prop-topic/cc-6th-percentages/v/fraction-decimal-and-percent-from-visual-model>

Other Resources needed

- Annex 1,2 and 3 (printed, one per group)
- A Training Room
- A White Board and markers
- Laptop
- Video Projector



ACTIVITY 2: Let's go Shopping

You are in a shopping mall during sales. There is a discount on selected items (see Annex 4). You have 800 euro to spend. You need to buy as many things as possible with this amount.

Step 1: Split the class into 4 groups

Step2: Provide each group with the items with their price tags and discount percentages (see Annex 4). Note that no calculators are allowed

Step3: Use a timer and provide 2 minutes to all groups in order to calculate the discounted prices of the items and decide which items to purchase. They can only purchase one of each item.

Step 4: After two minutes pass ask the groups to stop and present their results. The group which purchased the more items within the 800 euro budget wins.

Step 5: The winning team must present their strategy how they calculated the discounts and picked the right products

Step 6: The trainer presents the strategies for making quick calculations (mental calculations) without the need of a calculator. The trainer may show the video from <https://www.youtube.com/watch?v=YWHx0RsRNzI>

Resources for Trainer:

Strategies for quick calculations

<https://www.youtube.com/watch?v=YWHx0RsRNzI>

Other Resources needed

- Annex 4 (printed, one per group)
- A Timer
- A horn
- A Training Room
- A White Board and markers



Activity 3: Losing weight

You are on a diet. You are allowed to consume 10g of fat each day.

Step 1: Provide each group with a set of products (see other resources) or print out [Annex 5](#)

Step 2: Each group must calculate the number of grams that may be consumed from each product in order to get 10g of fat.

Step 3: Review the answers provided by each group

Step 4: Each group must present the method of their calculations to the rest of the class.

Resources for Trainer:

Strategies for quick calculations

<https://www.youtube.com/watch?v=YWHxORsRNzI>

Other Resources needed

- A Training Room
- A White Board and markers
- A chocolate, a nutrition bar, a packet of chips, a packet of biscuits that have with nutrition labels or print Annex 5 Products



Activity 4: The weak link

Everyone knows the game weak link. The Trainer asks each participant a question. If the participant gets it right, then he/she gets a 2nd question and so on. The trainer may allocate up to 45 sec to each player. Once time is up the trainer moves to the next player. For each right Question points are assigned to the player.

Step 1: Ask 4 volunteers to play the game.

Step 2: Print out the cards with the short questions. Start the game by asking the 1st player a question. Once the answer is provided move on to the next question until the 45 seconds are consumed. Move on to the next player

Step 3: Once all questions are answered calculate the points of each player. The one with the highest points wins.

Step 4: Review one by one each questions and provide the explanation and answers

Resources for Trainer:

Strategies for quick calculations

Other Resources needed

- A Training Room
- A White Board and markers



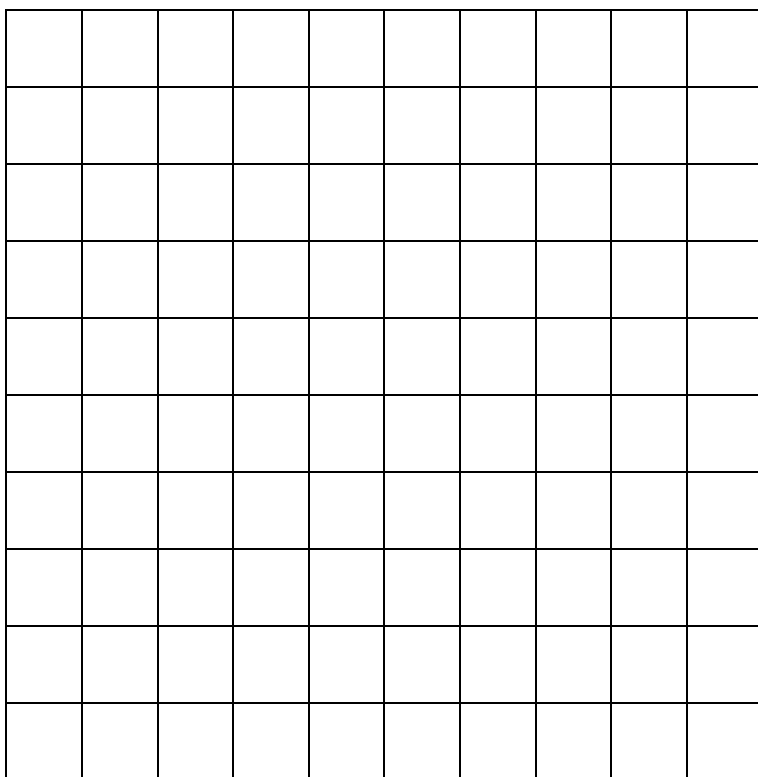
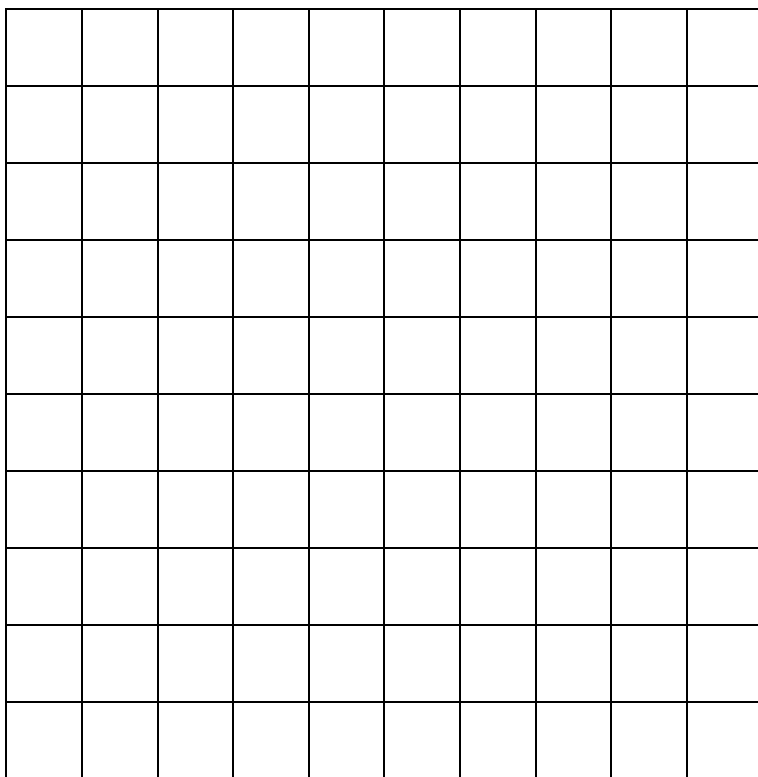
ANNEXES

Annex 1

1	2	3	4	5	6	7	8	9	10
2									
3									
4									
5									
6									
7									
8									
9									
10									



Annex 2:



Annex 3:



Express in

Percentage:

Fraction:

Decimal:



Załącznik nr 4:



On Offer!

Initial price: 500 euro

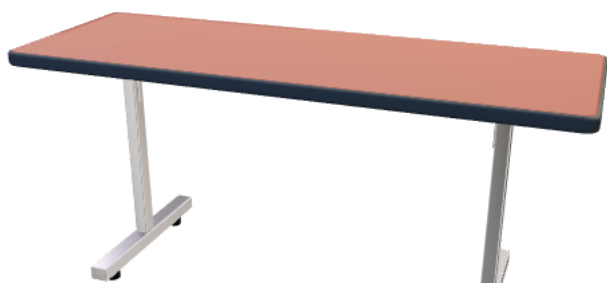
60% Discount



On Offer!

Initial price: 250 euro

20% Discount



On Offer!

Initial price: 125 euro

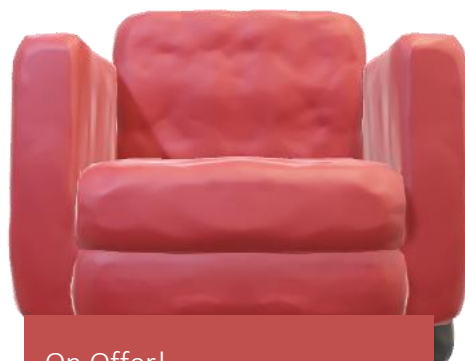
10% Discount



On Offer!

Initial price: 350 euro

50% Discount



On Offer!

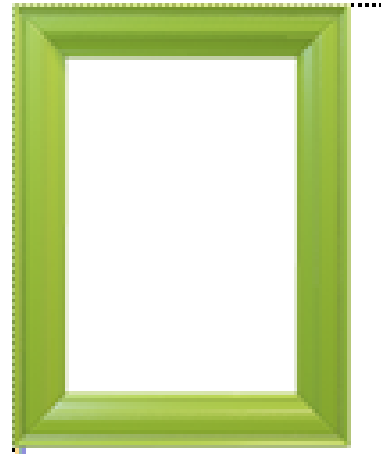
Initial price: 400 euro

30% Discount





On Offer!
Initial price: 100 euro
20% Discount



On Offer!
Initial price: 120 euro
10% Discount



On Offer!
Initial price: 200 euro
35% Discount



On Offer!
Initial price: 180 euro
30% Discount



Annex 5

1. Each Biscuit of 50 grams contains



How many Biscuits can you eat? _____

2. Each packet of chips of 35 grams contains



How many packets of chips can you eat? _____

3. Each bar of chocolate of 25 grams contains



How many bars of chocolate can you eat? _____

4. Each nutrition bar of 25 grams contains



How many nutrition bars can you eat? _____



Annex 6: Cards

Print the cards below, cut and fold. At the back of each card the answer is provided

Problem 1

The original price of a shirt was €20. It was decreased to €15 . What is the percent decrease of the price of this shirt.

Solution to Problem 1

Explanation of answer:

The absolute decrease is

$$20 - 15 = €5$$

The percent decrease is the absolute decrease divided by the the original price (part/whole).

$$\text{percent decrease} = 5 / 20 = 0.25$$

Multiply and divide 0.25 to obtain percent.

$$\begin{aligned} \text{percent decrease} &= 0.25 = 0.25 * 100 / 100 \\ &= 25 / 100 = 25\% \end{aligned}$$

Problem 2

Mary has a monthly salary of €1200. She spends €280 per month on food. What percent of her monthly salary does she spend on food?

Explanation of answer:

The part of her salary that is spent on food is €280 out of her monthly salary of €1200

$$\text{percent} = \text{part} / \text{whole} = 280 / 1200 = 0.23 \text{ (rounded to 2 decimal places)}$$

Multiply and divide 0.23 by 100 to convert in

$$\begin{aligned} \text{percent} &= 0.23 * 100 / 100 = 23 / 100 = \\ &= 23\% \end{aligned}$$



Problem 3

The price of a pair of trousers was decreased by 22% to €30. What was the original price of the trousers?

Solution to Problem 3

Let x be the original price and y be the absolute decrease. If the price was decreased to €30, then

$$x - y = 30$$

y is given by

$$y = 22\% \text{ of } x = (22 / 100) * x = 0.22 x$$

Substitute y by $0.22 x$ in the equation $x - y = 30$ and solve for x which the original price.

$$x - 0.22 x = 30$$

$$0.78 x = 30$$

$$x = €38.5$$

Check the solution to this problem by reducing the original price found €38.5 by 22% and see if it gives €30.

Problem 4

The price of an item changed from €120 to €100. Then later the price decreased again from €100 to €80. Which of the two decreases was larger in percentage term?

Explanation of answer:

First decrease in percent

$$\text{part / whole} = (120 - 100) / 120 = 0.17 = 17\%$$

Second decrease in percent

$$\text{part / whole} = (100 - 80) / 100 = 0.20 = 20\%$$

The second decrease was larger in percent term. The part were the same in both cases but the whole was smaller in the second decrease.



Problem 5

The price of an item decreased by 20% to €200. Then later the price decreased again from €200 to €150. What is the percent of decrease from the original price to the final price of €150?

Explanation of answer:

We first need to find the original price x .

The first decrease gives

$$x - 20\% x = 200$$

$$0.8x = 200$$

$$x = 200 / 0.8 = 250$$

The percentage decrease from the original price 250 to 150 is given by

$$\text{part / whole} = (250 - 150) / 250 = 0.4 = 40\%$$

Problem 6

A number increases from 30 to 40 and then decreases from 40 to 30. Compare the percent of increase from 30 to 40 and that of the decrease from 40 to 30.

Explanation of answer:

Percent increase from 30 to 40 is given by $(40 - 30) / 30 = 10 / 30 = 0.33 = 33\%$ (2 significant digits)

Percent decrease from 40 to 30 is given by $(40 - 30) / 40 = 0.25 = 25\%$

In absolute term, the percent decrease is less than the percent increase.

Problem 7

A family had dinner in a restaurant and paid €30 for food. They also had to pay 9.5% sale tax and 10% for the tip. How much did they pay for the dinner?

Explanation of answer:

They paid for food, sales tax and tip, hence

$$\text{total paid} = €30 + 9.5\% * 30 + 10\% * 30 = €35.85$$



Problem 8

A shop is offering discounts on shirts costing €20 each. If someone buys 2 shirts, he will be offered a discount of 15% on the first shirt and another 10% discount on the reduced price for the second shirt. How much would one pay for two shirts at this shop?

Explanation of answer:

The reduced price for the first shirt

$$20 - 15\% * 20 = €17$$

The reduced price for the second shirt.

The 10% discount will be on the already reduced price, hence the price of the second shirt is given by

$$17 - 10\% * 17 = €15.3$$

The total cost for the two shirts is

$$17 + 15.3 = €32.3$$

Problem 9

Smith invested €5000 for two years. For the first year, the rate of interest was 7% and the second year it was 8.5%. How much interest did he earn at the end of the two year period?

Explanation of answer:

Interest at the end of the first year

$$7\% * 5000 = €350$$

Interest at the end of the second year

$$8.5\% * (5000 + 350) = €454.75$$

Total interest at the end of the two year period is

$$€350 + €454.75 = €804.75$$



Problem 10

Janette invested €2000 at 5% compounded annually for 5 years. How much interest did she earn at the end of the 5 year period?

Explanation of answer:

At the of the first year, she has the principal plus the interest on the principal

$$P1 = 2000 + 5\% * 2000 = 2000(1 + 5\%)$$

At the of the second year, she has the principal P1 plus the interest on P1

$$P2 = P1 + 5\% * P1 = P1(1 + 5\%)$$

Substitute P1 by $2000(1 + 5\%)$ found above to find

$$P2 = 2000 * (1 + 5\%)^2$$

Continuing with this process, it can easily be shown that at the end of the 5th year, the principal is given by

$$P5 = 2000 * (1 + 5\%)^5$$

$$= 2000 * (1 + 0.05) = €2552.56$$

The interest earned at the end of 5 years is

$$€2552.56 - €2000 = €552.56$$

Problem 11

Tom borrowed €600 at 10% per year, simple interest, for 3 years. How much did he have to repay (principal + interest) at the end of the 3 year period?

Explanation of answer:

The interest to pay is given by

$$\text{Interest} = 600 * 10\% * 3 = €180$$

Total to repay

$$600 + 180 = €780$$



Problem 12

Out of a world population of approximately 6.6 billion, 1.2 billion people live in the richer countries of Europe, North America, Japan and Oceania and is growing at the rate of 0.25% per year, while the other 5.4 billion people live in the less developed countries and is growing at the rate of 1.5%. What will be the world population in 5 years if we assume that these rates of increase will stay constant for the next 5 years. (round answer to 3 significant digits)

Explanation of answer:

Let us first calculate the population PR in 5 years in the richer countries

$$PR = (1.2 + 0.25\% \cdot 1.2) = 1.2(1 + 0.25\%)$$

after one year

$$PR = 1.2(1 + 0.25\%) + 0.25\% \cdot 1.2(1 + 0.25\%)$$

$$= 1.2(1 + 0.25\%)^2 \text{ after two years}$$

Continue with the above and after 5 years, PR will be

$$PR = 1.2(1 + 0.25\%)^5 \text{ after 5 years}$$

Similar calculations can be used to find the population PL in less developed countries after 5 years.

$$PL = 5.4(1 + 1.5\%)^5 \text{ after 5 years}$$

The world population P after 5 years will be

$$P = PR + PL = 1.2(1 + 0.25\%)^5 + 5.4(1 + 1.5\%)^5 = 7.03 \text{ billion.}$$

Problem 13

Cassandra invested one part of her €10,000 at 7.5% per year and the other part at 8.5% per year. Her income from the two investment was €820. How much did she invest at each rate?

Explanation of answer:

Let x and y be the amount invested at 7.5% and 8.5% respectively

$$\text{Income} = €820 = 7.5\% \cdot x + 8.5\% \cdot y$$

The total amount invested is also known

$$10,000 = x + y$$

Solve the system of the equations to find x and y.

$$x = €3000 \text{ and } y = €7000$$

As a practice check that 7.5% of €3000 and 8.5% of €7000 gives €820.



Problem 14

The monthly salary S of a shop assistant is the sum of a fixed salary of €500 plus 5% of all monthly sales. What should the monthly sales be so that her monthly salary reaches €1500?

Explanation of answer:

Let S be the total monthly salary and x be the monthly sales, hence

$$S = 500 + 5\% \cdot x$$

Find sales x so that $S = 1500$, hence

$$1500 = 500 + 5\% \cdot x = 500 + 0.05x$$

Solve for x

$$x = (1500 - 500) / 0.05 = €20000$$

Problem 15

A chemist has a 20% and a 40% acid solutions. What amount of each solution should be used in order to make 300 ml of a 28% acid solution?

Explanation of answer:

Let x be the solution at 20% and y be the solution at 40%, hence

$$x + y = 300 \text{ ml}$$

We now write an equation that expresses that the total acid in the final 300 ml is equal to the sum of the amounts of acid in x and y

$$28\% \cdot 300 = 20\% \cdot x + 40\% \cdot y$$

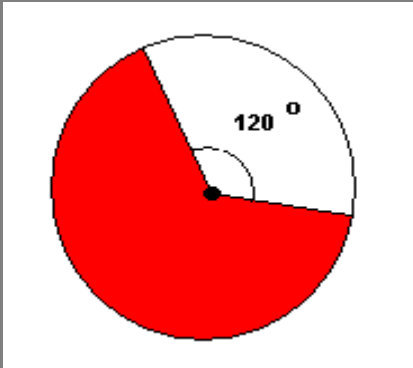
Solve the above system of equations to find

$$x = 180 \text{ and } y = 120$$



Problem 16

What percent of the total area of the circular disk is colored red?



Explanation of answer:

Total area of disk

$$A_d = \pi * r^2$$

Angle t in radians of central angle of red sector

$$t = (360 - 120) * \pi / 180 = (4/3) \pi$$

Area of red sector

$$A_s = (1/2) t * r^2$$

Percentage of total area in red

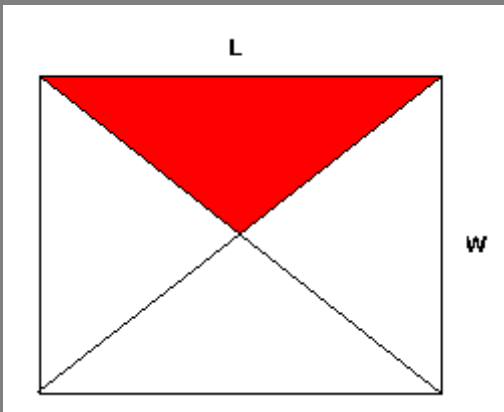
$$P = [(1/2) t * r^2] / [\pi * r^2]$$

$$= 4 / 6 = 66.7\% \text{ (3 significant digits)}$$

THINK: compare 66.7% to $240 / 360$, why are they equal?

Problem 17

What percent of the total area of the rectangle is colored red?



Explanation of answer:

Total area of rectangle

$$A_r = L * W$$

Area of triangle

$$A_t = (1/2) \text{ base } * \text{ height } = (1/2) [L * (1/2) W]$$

Percentage of area in red

$$P = (1/2) [L * (1/2) W] / [L * W] = 1/4 = 25\%$$



WORK AREA 2: MEASUREMENT &
CONVERSIONS

LEVEL 4

Workshop Title: Metric Units of Measurement and Conversions (Part 3)

CODE: L4.WA2.N2

LEVEL 4

WORK AREA 2 : MEASUREMENT & CONVERSIONS

PARTNER: ERIFO

Duration: 4-4,30 hours

Learning Outcomes Covered

WORK AREA 2 : MEASUREMENT & CONVERSIONS

This WS address the LO of the Work Area 2 referred to the AC ladder level 4.

In particular, the Unit 2.3 which addresses the way to convert numbers from one system to another system. It will help low skilled adults to achieve the useful skills to move from the AC ladder 4 to 5.

2.3 Converting numbers from one system to another system

LO1: Convert between **kilometer and miles**.

LO2: Convert between **kilograms and pounds**

LO3: Convert between **liter and gallons**.

LO4: Convert between different currencies

LO5: Convert from Celsius to Fahrenheit

Short introduction to the workshop

The Workshop is structured into 3 activities in order to provide specific knowledge and help low skilled adults in developing the ability to convert numbers from one system to another system. Having developed these skills, low-skilled adult will be able to move to AC ladder 5.

This workshop will address the following topics:

Activity 1: how to convert from Celsius to Fahrenheit Scales and viceversa

Activity 2: online converter tools (e.g. from liters to gallons)

Activity 3: online converter tools (e.g. for conversions among between different currencies)



Challenges during the implementation of activities and how can be addressed

This is an interactive WS which foresees the active involvement of participants. The diversification of the activities will guarantee major possibility to gain the learners' attention and interest. Moreover, the WS is based on a practical approach; learners will be involved in activities they could be asked to carry out in their everyday life; the connection with everyday life situations will ensure their participation and motivate them (eg. Conversion between currencies when they visit a foreign country). The use of the computer is an added value to diversify the activities and to learn the use of online tools.



ACTIVITY 1: At which temperature does the water boil?

Step 1: Overview about what Fahrenheit and Celsius Scale are. It would be helpful a thermometer with both scales.

Step 2: How to Convert from Fahrenheit to Celsius using the simple Formula $^{\circ}\text{C} = (^{\circ}\text{F} - 32) / 1.8$; learners will be involved in a written exercise; a sheet will be distributed where learners have to make conversions Celsius/Fahrenheit (Annex1); then the trainer can distribute another sheet (see <https://www.k5learning.com/worksheets/math/grade-3-temperature-weather-a.pdf>) with pictures and temperatures associated to them; the temperatures will be expressed in Fahrenheit. (e.g. Ice = 32°F , how many $^{\circ}\text{C}$ degrees do 32°F correspond to?)

$$32^{\circ}\text{F} - 32 = 0$$

$$0 / 1.8 = 0$$

Thus, $32^{\circ}\text{F} = 0^{\circ}\text{C}$ which is the temperature that turns water into ice)

Step 3: How to Convert from Celsius to Fahrenheit. Using the simple Formula $^{\circ}\text{F} = (^{\circ}\text{C} * 1.8) + 32$, learners will be asked to complete a written exercise in which they have to convert temperatures expressed in $^{\circ}\text{C}$ in Fahrenheit (<https://www.k5learning.com/worksheets/math/grade-3-temperature-weather-b.pdf>).

(e.g. Boiling water = 100°C , how many $^{\circ}\text{F}$ do 100°C correspond to?)

$$100^{\circ}\text{C} * 1.8 = 180$$

$$180 + 32 = 212^{\circ}\text{F}$$

Thus, $100^{\circ}\text{C} = 212^{\circ}\text{F}$ which is the temperature of boiling water)

The trainer can also develop a working sheet including pictures representing different cities and their temperatures in a specific time of the year during which the workshop will take place so that learners will also gain an understanding of differences in temperatures in different places and regions.

Resources for Trainer

- <https://www.k5learning.com/worksheets/math/grade-3-temperature-weather-a.pdf>
- <https://www.k5learning.com/worksheets/math/grade-3-temperature-weather-b.pdf>



- | |
|---|
| <ul style="list-style-type: none">• Thermometer (both Celsius and Fahrenheit Scales)• Working sheet for °C/F conversions• Calculators |
| Other Resources <ul style="list-style-type: none">• Working sheets• Training room equipped with computers |



ACTIVITY 2: A pint of beer please!

Step 1: the mentor will make an introduction on liter and gallons; he/she will explain how many liters 1 gallon correspond to and also clarify where this different unit of capacity is used. The mentor has to show how to use online tools to convert from one system to another one (online converter from liter to gallons and vice versa <https://www.unitconverters.net/volume/liters-to-gallons.htm>)

Step 2: learners have to test what they learnt by using a computer

Resources for Trainer

- <https://www.unitconverters.net/volume/liters-to-gallons.htm>
- smartboard

Other Resources

- Training room equipped with computers
- Smart board



ACTIVITY 3: On the road... with money

Step 1: the mentor will make an introduction on the value of different currencies (euro, dollar...); he/she will explain, for example, how many euros 1 dollar correspond to and also clarify where this currency is used. The mentor has to show how to use online tools to convert from one currency to another one (online converter <http://www.convertmymoney.com/>)

Step 2: the mentor provides learners with a scenario (eg. At the restaurant in the US) and learners have to make conversions based on the scenario

Resources for Trainer

- Working sheet for conversions
- Scenarios descriptions
- Computers
- <http://www.convertmymoney.com/>

Other Resources

- Training room with PCs



ANNEXES

Annex 1

Name : _____ Score : _____

Teacher : _____ Date : _____

Converting Fahrenheit and Celsius

Convert Fahrenheit to Celsius

1) 39°F _____

5) 94°F _____

2) 83°F _____

6) 111°F _____

3) 65°F _____

7) 34°F _____

4) 62°F _____

8) 38°F _____

Convert Celsius to Fahrenheit

9) 44°C _____

13) 45°C _____

10) 38°C _____

14) 12°C _____

11) 9°C _____

15) 23°C _____

12) 51°C _____

16) 21°C _____



WORK AREA 3: SHAPES & SPACE

LEVEL 4

Workshop Title: “Exploring Maths within a world of Shapes”

CODE: L4.WA3.N3

LEVEL 4

WORK AREA 3: SHAPES & SPACE

PARTNER: DIMITRA

Duration: 5-7 hours

Learning Outcomes Covered:

WORK AREA 3: SHAPES & SPACE

Unit 3.2 Basic Geometric Calculations

LO1: Identify the appropriate formulas to calculate area, perimeter and volume of 3D Shapes

LO2: Calculate the area, perimeter and volume of 3D shapes by using the appropriate formulas

Unit 3.4 Space

LO1: Define the coordinates on an axis

LO2: Explain the coordinates in a space

LO3: Illustrate by example how to locate objects from one plane to another

LO4: Illustrate by example how to locate objects on an axis

LO5: Illustrate by example how to locate objects in a space

LO6: Identify the steps for calculating the distance between points

LO7: Measure distance between points

Short introduction to the workshop

The workshop focuses on describing and implementing the basic 3d geometry calculations. The activities include: short presentation of basic geometric



calculations., provision of formulas and guidance on how to use 3d geometric calculations, practical exercise to fully understand and implement the presented formulas. The workshop also focuses on learning activities referring to the Cartesian Coordinate system. The activities include: short description of the Cartesian system, creation of a 2d and 3D Cartesian coordinate system by trainees using innovative techniques (boxes for 3D system), short analysis and specification of planes in 3 dimensions, measurement of distance between 2 points through specific formula.

How the workshop (its activities) teach the LOs to the participants

The workshop for basic 3d geometric calculations not only provides a substantial theoretical framework for the trainees but also presents a series of practical activities connected to it. In that way, the trainee will have the opportunity to combine his/her theoretical knowledge from the first activities with practical exercises, putting his/her just obtained knowledge in action. In addition, through the use of things that are familiar to them and their everyday life, the workshop can enrich their knowledge in more complicated subjects such as defining space through the Cartesian Coordinate system and generally enhance their spatial awareness.

Challenges during the implementation of activities and how can be addressed

This workshop could be quite challenging for people that have a low scaled mathematical thinking ability. In order to moderate this problem, the workshop includes a step by step educational procedure that begins with easier calculations and continues with more advanced so as to be easy for the trainee.



ACTIVITY 1: Introduction to 3d geometric calculations.

STEP 1:

Short presentation of basic 3d geometric calculations.

Resources for Trainer

The Organic Chemistry Tutor, 3D Shapes - Faces, Edges, and Vertices - Euler's Formula – Geometry <https://www.youtube.com/watch?v=Wmwe1fmR1SM> Provision of exercise sheet. (accessed on 20/9/2019)

Slideshare Net, 3D Geometry Formulas eBook

<https://www.slideshare.net/PDF-eBooks-For-Free/geometry-formulas-2d-and-3d-ebook> (accessed on 20/9/2019)

Other Resources

- A classroom equipped with a computer for PPP presentations.



ACTIVITY 2: Liven up the 3d Math forms

STEP 1: Provide specific guidance on how to use 3d geometric calculations.

STEP 2: Put different pieces of papers with written formulas on them in a ball and go around and ask each of the participants to pick one. Then ask them to find out which object is measured using this formula and ask them to build it out of material you have provided (paper, straws, legos) Tip: if you see that this is easy for them do more rounds.

Resources for Trainer

The Organic Chemistry Tutor, 3D Shapes - Faces, Edges, and Vertices - Euler's Formula – Geometry <https://www.youtube.com/watch?v=Wmwe1fmR1SM> Provision of exercise sheet. (accessed on 20/9/2019)

Slideshare Net, 3D Geometry Formulas eBook

<https://www.slideshare.net/PDF-eBooks-For-Free/geometry-formulas-2d-and-3d-ebook> (accessed on 20/9/2019)

Other Resources

- A training room or an outdoor space.
- Books, pictures, Notebooks, straws, tape, paper, lego, etc.



ACTIVITY 3: 2D or 3D? All together!

STEP 1: Following the previous activities ask them to exchange between them the different objects they have designed or created in activities 2 and 3 and calculate the areas and perimeters. This activity requires a deep understanding of all formulas either 2D or 3D and ability to distinguish among them. Ask them to note them down and then present the results to the classroom along with their work from the previous step.

Resources for Trainer

The Organic Chemistry Tutor, 3D Shapes - Faces, Edges, and Vertices - Euler's Formula – Geometry <https://www.youtube.com/watch?v=Wmwe1fmR1SM> Provision of exercise sheet. (accessed on 20/9/2019)

Slideshare Net, 3D Geometry Formulas eBook

<https://www.slideshare.net/PDF-eBooks-For-Free/geometry-formulas-2d-and-3d-ebook> (accessed on 20/9/2019)

Other Resources

- A training room or an outdoor space.
- Books, pictures, Notebooks, straws, tape, paper, lego, etc.



ACTIVITY 4: Introduction to the Cartesian System

STEP 1:

Present a short description of the Cartesian system and then ask the trainees to identify the coordinates in an axis.

STEP 2:

Ask them to draw a 2d Cartesian coordinate system with the number lines filled in (eg. dm, cm, or just undefined equal spaces).

STEP 3:

Ask them to close their eyes and point out randomly a point with their finger. After that, ask them to find its place on the coordinate system.

Resources for Trainer

- PPTs for presenting the material

Other Resources

- A training room or an outdoor space.
- A4 papers or quadrilateral pieces of carton, ruler, rubbers or pencil sharpener, carton boxes, Cartesian graph paper



ACTIVITY 5: Let's explore the coordinates

STEP 1: Explain the coordinates in space and ask the trainees to identify the three coordinates (x,y,z).

STEP 2: With the use of a box and a ruler, ask them to create a 3d Cartesian coordinate system.

STEP 3: Choose a small object like a rubber or a pencil sharpener and locate it on the 3d Cartesian coordinate system they have just created.

Resources for Trainer

- PPTs for presenting the material

Other Resources needed

- A training room or an outdoor space.
- A4 papers or quadrilateral pieces of carton, ruler, rubbers or pencil sharpener, carton boxes, Cartesian graph paper



ACTIVITY 6: Planes and dimensions

STEP 1: Specify planes in 3 dimensions. Explain the way each plane is defined (points, lines) and give examples.

STEP 2: With the use of A4 paper or a piece of carton in the shape of a simple quadrilateral, ask the trainees to define a plane and explain their concept of its definition (e.g. how many points we need, what kind of restrictions we have pointwise?)

Resources for Trainer:

- PPTs for presenting the material

Other Resources needed

- A training room or an outdoor space.
- A4 papers or quadrilateral pieces of carton, ruler, rubbers or pencil sharpener, carton boxes, Cartesian graph paper



ACTIVITY 7: Keeping the distance

STEP 1:

Presentation of the distance formula and implementation on the Cartesian system.

STEP 2:

According to the presented theoretical framework, ask trainees to measure distance between 2 points through the distance formula given. The activity will be implemented on a Cartesian graph paper.

Resources for Trainer:

- PPTs for presenting the material

Other Resources needed:

- A training room or an outdoor space.
- A4 papers or quadrilateral pieces of carton, ruler, rubbers or pencil sharpener, carton boxes, Cartesian graph paper



LEVEL 4

WORK AREA 4: HANDLING INFORMATION

Workshop Title: Easy Decision (If You Know What You're Talking About)!

CODE: L4.WA4.N4

LEVEL 4

WORLD AREA 4: HANDLING INFORMATION

PARTNER: MET

Duration: 5 hours

Learning Outcomes Covered

WORLD AREA 4: HANDLING INFORMATION

Unit 4.1 Visualizing Data

LO1: Interpret charts to draw conclusions:

- Explain the information and data illustrated in a chart
- Analyze the data
- Deduce the core information

LO2: Create charts/graphs when appropriate:

- Select the most relevant data
- Organize data
- Show data in a chart/graph
- Highlight the most relevant data

Unit 4.2 Probabilities & interpretation

LO1: Participants will be able to: Have knowledge of the basic statistical concepts:

- probability : Reading probabilities, using fractions, decimals and percentages: a) Classify probabilities as fractions or decimals from 0 to 1, b) express into % c) interpret outcomes which are 0 or 1
- Frequency
- mean/average in a set of numbers
- median in a list of numbers

LO2: Be able to calculate

- probability: Quantify probabilities, using fractions, decimals and percentages: a)



Classify probabilities as fractions or decimals from 0 to 1, b)express into % c) interpret outcomes which are 0 or 1

- Frequency
- mean/average in a set of numbers
- median in a list of numbers

LO3: Understanding Population and Sampling

LO4: Differentiate between quantitative and qualitative

- datahttp://www.analyzemath.com/statistics/introduction_statistics.html

LO5: Use tables or diagrams to collect, display and analyze data:

- Organize data in tables or diagrams for basic analysis

LO6: Interpret a given probability:

- probability
- Frequency
- mean/average in a set of numbers
- median in a list of numbers

Short introduction to the workshop

This workshop aims to enable the learners to interpret charts and draw conclusions. Moreover, they will learn to decide which visualization is appropriate for what kind of data and how to create a chart, diagram etc.

The workshop's second topic offers the learners a good understanding of what probabilities are and familiarizes them with classical statistical concepts. Based on that knowledge they should acquire the skills to calculate probabilities with given information. Moreover, they gain knowledge of how to organize data in tables or diagrams for basic analyses.

To train the new knowledge and skills, the learners will solve simple problems, e.g. identifying the relevant/effective elements when making a decision, discussing facts, risks, opportunities of choices, or applying their knowledge of data visualization, either by explaining a given chart or illustration or by creating illustrations/charts of their own.

How the workshop activities teach the LOs to the participants



The contents of this workshop are conveyed through various formats. These include frontal teaching, in which the teacher first introduces the individual topics. Basic connections are described. Learners have the opportunity to ask questions at the beginning of a thematic field.

This perspective is supplemented by corresponding explanatory videos. These videos can be watched and played individually by the learners depending on their technical equipment. This allows them to determine themselves when to press e.g. the stop button or watch individual sections repeatedly.

Additionally, there are online tests and games for some areas. This allows the learners to apply their newly acquired knowledge directly. This makes it quickly clear at which points content has not yet been understood. Since the teacher is always present, they can offer direct support and explain certain things again.

Challenges during the implementation of activities and how can be addressed

For some activities it is important that the previous workshop has already been completed and understood, as contents build on each other. Additional time should therefore be planned to repeat the previous content if necessary, if it has not yet been sufficiently understood.



ACTIVITY 1: Frequencies, averages and the median using age as an example

Step 1: In the first step, all participants in the group tell their age. One writes down the ages on a board (see table in [Annex 1](#)).

Step 2: Together they calculate the average age of the group. To this they add up all the numbers. Then they divide the result by the number of people (see table in [Annex 1](#)).

Step 3: According to this, age categories are to be created. The age groups serve to better illustrate the allocation.

For example, the following groups could be formed: Under 30 years, the age group 31-45 years, the age group 46-60 years, etc (see example in [Annex 2](#)).

The age groups should be chosen so that not all persons are in the same group. If necessary, age groups should be narrower, e.g. 15-20 years, 21-26 years, 27-32 years, etc.

Step 4: Frequency is the number of times a value occurs in a set of data.

For this purpose, the number of persons assigned to each age group should be counted. This is then the absolute frequency.

Step 5: Next, the relative frequency will be addressed. Relative frequency is the proportion of absolute frequency in the total number of observations.

Discuss in the group what it means when the result is close to 0 and what it means when the result is close to 1. Here in the example the lowest value is $3/25$, i.e. 0.12. This means that the fewest Persons are in this age group. The highest relative frequency is $10/25$, which is 0.4, meaning that most people in this age group are in this age group.

Step 6: Percentage frequency is the relative frequency expressed as a percentage.

In other words, dividing the absolute frequency by the total number of observations.

Now calculate the percentage frequencies of persons per age group (see [Annex 2](#)).

Step 7: After that, they will also calculate the median and get to know the difference. Watch video 1 (see link under Resources for Trainer)



To practice these calculations, further examples can be chosen. These should be calculated by the learners themselves. The results can then be compared in the group.

For example, they can additionally count how many persons live in each of their households and calculate frequencies, averages and the median.

Resources for Trainer

- Annex 1, 2, 3
- Video 1: <https://www.khanacademy.org/math/ap-statistics/summarizing-quantitative-data-ap/measuring-center-quantitative/v/mean-median-and-mode>

Other Resources needed

- Annex 1,2,3 (printed for each learner)
- A Training Room
- A White Board and markers
- Possibility to play video (smartboard, beamer or several tablets)



ACTIVITY 2: Which data fits in which diagram

The main functions of a diagram are to present data in a clear and concise manner and to arouse interest in further exploring the contents of the diagram.

Charts are used in situations where a simple table does not adequately illustrate important relationships or patterns between data points.

Remember the explanations on pie charts and bar charts from the previous workshop. Now it's about displaying the data from Activity 1 in charts.

Step 1: Now we use the data from the table ([Annex 2](#)).

- (If you did not create the table on your computer, open an Excel or Word document and create the same table there.)
- Now, select the data for which you want to create a chart.
- Click INSERT > Recommended Charts.
- On the Recommended Charts tab, scroll through the list of charts that Excel recommends for your data, and click any chart to see how your data will look. If you don't see a chart you like, click All Charts to see all the available chart types (e.g. pie chart or bar chart).
- When you find the chart you like, click it > OK.
- Use the Chart Elements, Chart Styles, and Chart Filters buttons, next to the upper-right corner of the chart to add chart elements like axis titles or data labels, customize the look of your chart, or change the data that is shown in the chart.
- To access additional design and formatting features, click anywhere in the chart to add the CHART TOOLS to the ribbon, and then click the options you want on the DESIGN and FORMAT tabs.

Step 2: Compare your charts with those in [Annex 4](#) and see if there are differences and what might be missing in your chart.

Step 3: Create new figures or even new categories (e.g. number of participants with brown eyes, green eyes, blue eyes, etc.) Change this data in your table or create a new table. Use this table to create a new chart.

Step 4: Watch Video 2 on how to describe Bar charts.



Step 5: Present your diagram to the other participants. Explain the information and data illustrated in a chart. Deduce the core information.

Resources for Trainer

- Annex 2,4
- Video 2: <https://www.youtube.com/watch?v=QC8vN3GVQhI>

Other Resources needed

- A PC or tablet with an office program, ideally for each learner or for small groups of 2-3 people
- Annexes 2 and 4 as copies or visible on a wall (smartboard, beamer, projector, etc.)



ACTIVITY 3: Basic statistical concepts such as probability

Statistics is a field of mathematics. Statistics includes the methods for collecting, organizing and analyzing data. From this, meaningful conclusions are then drawn.

Probabilities are associated with trials where the outcome is not known in advance or cannot be predicted. For example, if you flip a coin, do you get a head or a tail? If you roll a die, do you get 1, 2, 3, 4, 5 or 6?

Probability indicates how likely an event related to this type of attempt will be and quantifies it. The value of a probability is a number between 0 and 1. An event that cannot occur has a probability (of occurrence) equal to 0 and the probability of an event that is certain to occur has a probability of 1. This has already been discussed in Activity 1.

Step 1: Trainer explains probabilities, Samples and populations:

To quantify the probabilities, we must define the sample space of an experiment and the events that can be associated with that experiment.

The sample space is the set of all possible results in an experiment.

For Example, if a die is rolled, the sample space S is given by

$$S = \{1, 2, 3, 4, 5, 6\}$$

We define an event as a certain result of an experiment. An event is a subset of the sample space.

For example, a die is rolled. Define the event E as the set of possible results where the number on the face of the dice is even. The event E is given by

$$E = \{2, 4, 6\}$$

A population often consists of a large group of specifically defined elements. For example, the population of a particular country means all people living within the borders of that country.

It is usually not possible or useful to measure data for each element of the population under study. We randomly select a small group of elements from the population and call it a sample. On the basis of several samples we then draw conclusions about the population.



So, what is the difference between sample and population.

The population is the entire group of objects, individuals, or events.

A sample is part of the population that is chosen to represent the entire group

Step 2: Watch Video 3.

Step 3: Each Learner plays the online game on Population and Sample.

Step 4: Trainer explains difference between qualitative and quantitative data:

Data are quantitative if the observations or measurements on a particular variable in a sample or population have numerical values, i.e. they can be counted.

For example: height, weight, number of children, blood pressure, current, voltage.

Data are qualitative when words, groups and categories represent the observations or measurements.

Example: colors, yes-no answers, blood group.

Step 5: Each Learner plays the online multiple-choice test.

Resources for Trainer

- Video 3: <https://www.youtube.com/watch?v=eIZD1BFfw8E>
- Online game: quizizz.com/join/quiz/572b3a0346b0b212119754ec/start
- Online multiple Choice test:
[https://reviewgamezone.com/mc/candidate/test/
test_id=19755&title=Qualitative%20And%20Quantitative%20Data](https://reviewgamezone.com/mc/candidate/test/test_id=19755&title=Qualitative%20And%20Quantitative%20Data)

Other Resources needed

- A PC or tablet with an office program, ideally for each learner or for small groups of 2-3 people
- Possibility to play video (smartboard, beamer or several tablets)



ANNEXES

Annex 1

Name	Age
Sum of all ages:	
Average age (sum of all ages divided by the number of persons):	



Annex 2 (as an example calculation)

Age Group	Number of persons in this age group (tally sheet)	Absolute frequency (Total number of all persons in this age group)	Relative frequency	Percentage frequency of persons per age group (The absolute frequency divided by the total number of persons per age group, then multiplied by 100)
Under 30	IIII	3	3 25	$(3/25)*100 = 12\%$
30-40	IIII IIII	10	10 25	40%
41-50	IIII	4	4 25	16%
51 and older	IIII III	8	8 25	32%
Total number	IIII IIII IIII IIII IIII	25	25 25	100%



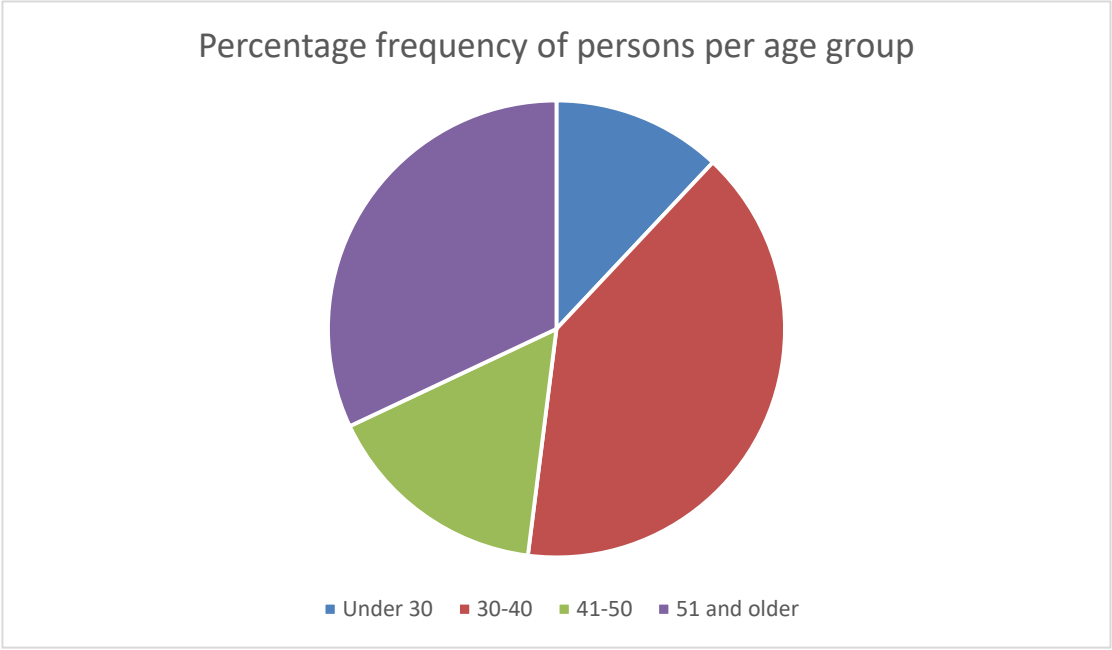
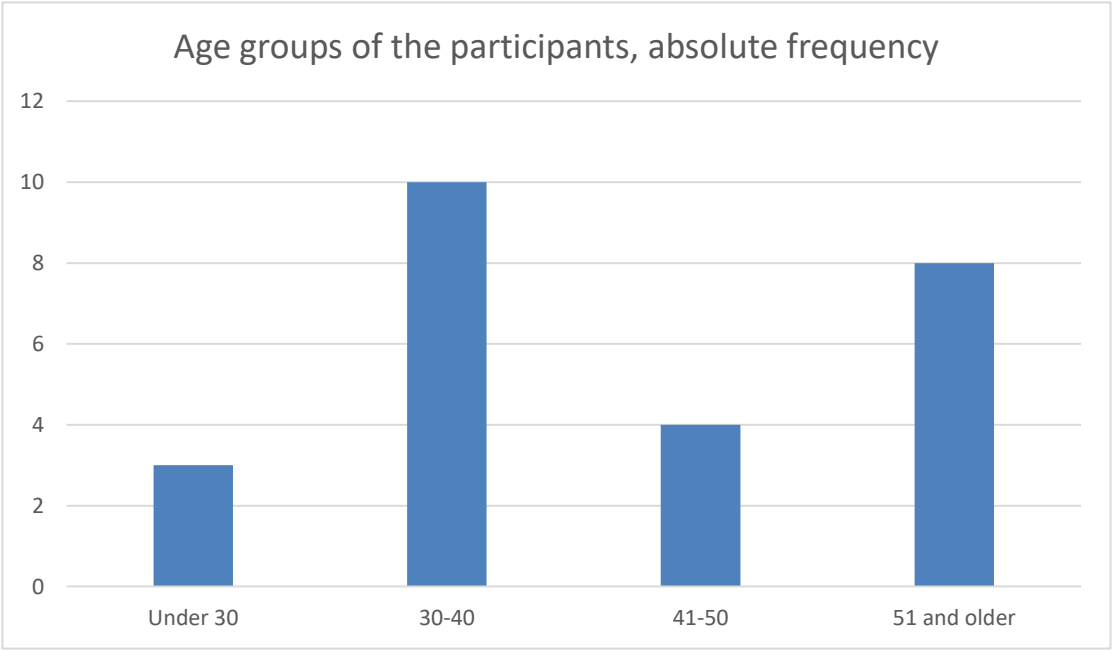
Annex 3

(as an example calculation)

Group	Number of persons in this group (tally sheet)	Absolute frequency (Total number of all persons in this group)	Relative frequency	Proportion of persons in this group in % (The absolute frequency divided by the total number of persons per group, then multiplied by 100)
Group 1				
Group 2				
Group 3				
(further groups to be added)				
Total number			55	



Annex 4



Active Citizenship Level 5

LEVEL 5

WORK AREA 1: NUMBERS

Workshop Title: Working with Exponents
CODE: L5.WA1.N1
LEVEL 5
WORL AREA 4: NUMBERS
PARTNER: EDITC
Duration: 5 hours
Learning Outcomes Covered
WORL AREA 1: NUMBERS Unit 1.10 1.10 Rules of Order in Calculations LO1 Exponents: Have the knowledge of the order of operations to solve expressions and equations LO2 Have the knowledge of the order of operations to solve expressions and equations with Exponents in Multiplication and division LO3 Have the knowledge of the order of operations to solve expressions and equations with Exponents in Addition and subtraction LO4 Apply the order of operations when solving an expression or equation with Exponents
Short introduction to the workshop
Exponents are an essential part of algebra, polynomial equations and higher-level math courses, but many adults struggle to understand how to work with them. Before you start teaching your students how to multiply exponents, you might want to do a quick review with them on the basics of how exponents work. (See Annex 1) Exponents (also called powers) are governed by rules, like everything else in math class. Here’s a quick recap: An exponent is a way of expressing repeated multiplication. For example, 3^5 represents three multiplied by itself five times: $3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$ $3^5 = 243$ The first number is referred to as the base. It represents the number that gets multiplied. The second, smaller number is the exponent. It represents the number of times the base is multiplied by itself.



How the workshop activities teach the LOs to the participants

This workshop begins with an introduction to the 7 rules for exponents followed by 2 activities:

Exponents War: This activity is a game with cards where the participants will practice the calculation of exponents.

Exponents Hunt: This is a game where participants apply the 7 rules with an innovative game that involves the application of the 7 rules for exponents. Some sample exercises (cards to be used for the game) are provided in Annex 2 along with step by step instructions with the solutions.

Upon the completion of this workshop, students should be familiar with exponents and in position to solve simple exponent problems.

Challenges during the implementation of activities and how can be addressed

Exponents for some people are difficult to comprehend. It is crucial that examples are provided prior the 2 games explaining the 7 rules. It is recommended that 45 minutes are allocated for the introduction of the 7 rules and for practical examples where exponents are used and how exponent questions could be solved. The trainer may use examples from Annex 2.



Step by step instructions for the trainers/mentors

ACTIVITY 1: Exponent War

A classic card game — but with an exponentially fun twist!

Step 1: Before starting the Exponent War game start with a revision on Exponent Rules (see Annex 1)

Step 2: Students are split in teams of two and face off against another pair. Give each team a deck of cards (with the queens, jacks, and kings taken out) and have each player pull out two cards. The first card is the base, and the second card is the exponent.

Step 3: Each pair has to race to solve their equation and find the product. The team with the highest answer wins. Set a timer for the class and see who can rack up the most points.

While students are playing, walk around the classroom and make sure they're not missing any steps. If you see a lot of mistakes or struggling students, take it as a sign that you might need to do some review.

Resources for Trainer

- Annex 1
- <https://www.mesacc.edu/~scotz47781/mat120/notes/exponents/review/review.html>

Other Resources needed

- A Training Room
- A White Board and markers
- 2 Decks of cards



Step by step instructions for the trainers/mentors

ACTIVITY 2: Exponent Hunts

Step 1: Give your students a chance to search for treasure and explore the classroom with an exponent scavenger hunt.

Step 2: Divide your class into groups of three or four.

Step 3: Depending on the number of groups you have, make several different sets of cards. Start each set with a card that has a problem on it. Write the answer to the problem on the next card, and put another problem on the back. Keep going until you have three or four sets of problems (or more). (see Annex 2 for examples). The first card is given to the students the remaining cards are placed in the room in different places on walls, doors, windows, desks etc with the answer visible on the front and another problem to solve on the back.

Step 4: Starting with the first card, each group must solve the problem and find the correct answer. Then they need to pick up the card showing the correct answer and proceed with the next problem (at the back of the card). When they find the correct answer (i.e. pick up the correct card), they can flip it over and solve the next problem. Give students scrap paper for solving, and let them start hunting for their answers. Whichever team finishes first is the winner!

Resources for Trainer

- Annex 2

Other Resources needed

- A Training Room
- A White Board and markers
- Scrap papers and pencils



ANNEXES

Annex 1

1. **Product of powers rule:** Add powers together when multiplying like bases
2. **Quotient of powers rule:** Subtract powers when dividing like bases
3. **Power of powers rule:** Multiply powers together when raising a power by another exponent
4. **Zero power rule:** Any base raised to the power of zero becomes one
5. **Negative exponent rule:** To change a negative exponent to a positive one, flip it into a reciprocal
6. **Power of a product rule:** Distribute power to each base when raising several variables by a power
7. **Power of a quotient rule:** Distribute power to all values in a quotient

Product of Powers Rule: $a^m \cdot a^n = a^{m+n}$, this says that to multiply two exponents with the same base, you keep the base and add the powers.	$x \cdot x^5 = x^6$ $y^4 \cdot y^9 = y^{13}$
$\frac{a^m}{a^n} = a^{m-n}$ Quotient Rule: $\frac{a^m}{a^n} = a^{m-n}$, this says that to divide two exponents with the same base, you keep the base and subtract the powers. This is similar to reducing fractions; when you subtract the powers put the answer in the numerator or denominator depending on where the higher power was located. If the higher power is in the denominator, put the difference in the denominator and vice versa, this will help avoid negative exponents.	$\frac{x^5}{x^3} = x^2$ $\frac{y^4}{y^9} = \frac{1}{y^5}$ $\frac{x^3 y^2}{x^2 y^5} = \frac{x}{y^3}$
Power of Powers Rule (Powers to Powers): $(a^m)^n = a^{mn}$, this says that to raise a power to a power you need to multiply the exponents. There are several other rules that go along with the power rule, such as the product-to-powers rule and the quotient-to-powers rule.	$(x^5)^4 = x^{20}$ $(2x^4 y^2)^3 = 2^3 x^{12} y^6 = 8x^{12} y^6$ $\left(\frac{x^2}{y^5}\right)^4 = \frac{x^4}{y^{20}}$
Zero-Exponent Rule: $a^0 = 1$, this says that anything raised to the zero power is 1.	$3^0 = 1$ $(5x^3 y^4)^0 = 1$



Negative Exponent Rule: $a^{-n} = \frac{1}{a^n}$, this says that negative exponents in the numerator get moved to the denominator and become positive exponents. Negative exponents in the denominator get moved to the numerator and become positive exponents. Only move the negative exponents.	$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$ $4x^{-2} = \frac{4}{x^2}$ $\frac{x^{-3}}{y^{-7}} = \frac{y^7}{x^3}$
The Power of a Product rule states that a term raised to a power is equal to the product of its factors raised to the same power	$3^3 \cdot 2^3 = (3 \cdot 2)^3$
Power of a quotient rule: Distribute power to all values in a quotient	$3^3 / 2^3 = (3/2)^3$



Annex 2

Card	Front (Question)	Back (Answer of Previous Card)
1	Simplify $\frac{(3x^4y^{-2})^{-3}}{(2x^3y^2)^{-2}}$	
2	Simplify: $\frac{(3x^{-5}y^2)^0}{(4x^{-3}y^2)^{-2}}$	$\frac{4y^{10}}{27x^{10}}$
3	Simplify: $\left(\frac{3x^3y^{-2}}{4x^5y^{-3}}\right)^{-3}$	$\frac{16y^4}{x^6}$
4	Simplify: $\frac{(-4x^3y^{-1})^2(5x^3y^{-2})^0}{(2x^4y)^3}$	$\frac{64x^6}{27y^3}$
		$\frac{16}{8x^6y^5} = \frac{2}{x^6y^5}$



Example 1 – Simplify: $\frac{(3x^4y^{-2})^{-3}}{(2x^3y^2)^{-2}}$

Step 1: Apply the Zero-Exponent Rule. In this case, there are no zero powers.	$\frac{(3x^4y^{-2})^{-3}}{(2x^3y^2)^{-2}}$
Step 2: Apply the Power Rule.	$\frac{3^{-3}x^{-12}y^6}{2^{-2}x^{-2}y^{-4}}$
Step 3: Apply the Negative Exponent Rule. Move every negative exponent in the numerator to the denominator and vice versa.	$\frac{y^6 2^2 x^2 y^4}{3^3 x^{12}}$
Step 4: Apply the Product Rule.	$\frac{2^2 x^2 y^{10}}{3^3 x^{12}}$
Step 5: Apply the Quotient Rule. In this case, the x's ended up in the denominator because there were 10 more x's in the denominator.	$\frac{2^2 y^{10}}{3^3 x^{10}}$
Step 6: Raise each coefficient (or number) to the appropriate power and then simplify or reduce any remaining fractions. In this case, the fraction does not reduce.	$\frac{4y^{10}}{27x^{10}}$



Example 2 –Simplify: $\frac{(3x^{-5}y^2)^0}{(4x^{-3}y^2)^{-2}}$

Step 1: Apply the Zero-Exponent Rule.	$\frac{1}{(4x^{-3}y^2)^{-2}}$
Step 2: Apply the Power Rule.	$\frac{1}{4^{-2}x^6y^{-4}}$
Step 3: Apply the Negative Exponent Rule. Move every negative exponent in the numerator to the denominator and vice versa.	$\frac{4^2y^4}{x^6}$
Step 4: Apply the Product Rule. In this case, the product rule does not apply.	$\frac{4^2y^4}{x^6}$
Step 5: Apply the Quotient Rule. In this case, the quotient rule does not apply.	$\frac{4^2y^4}{x^6}$
Step 6: Raise each coefficient (or number) to the appropriate power and then simplify or reduce any remaining fractions. In this case, the fraction does not reduce.	$\frac{16y^4}{x^6}$



Example 3 –Simplify: $\left(\frac{3x^3y^{-2}}{4x^5y^{-3}}\right)^{-3}$

Step 1: Apply the Zero-Exponent Rule. In this case, there are no zero powers.	$\left(\frac{3x^3y^{-2}}{4x^5y^{-3}}\right)^{-3}$
Step 2: Apply the Power Rule.	$\frac{3^{-3}x^{-9}y^6}{4^{-3}x^{-15}y^9}$
Step 3: Apply the Negative Exponent Rule. Move every negative exponent in the numerator to the denominator and vice versa.	$\frac{y^6 4^3 x^{15}}{3^3 x^9 y^9}$
Step 4: Apply the Product Rule. In this case, the product rule does not apply.	$\frac{y^6 4^3 x^{15}}{3^3 x^9 y^9}$
Step 5: Apply the Quotient Rule. In this case, the x's ended up in the numerator and the y's ended up in the denominator.	$\frac{4^3 x^6}{3^3 y^3}$
Step 6: Raise each coefficient (or number) to the appropriate power and then simplify or reduce any remaining fractions. In this case, the fraction does not reduce.	$\frac{64x^6}{27y^3}$



Example 4 –Simplify: $\frac{(-4x^3y^{-1})^2(5x^3y^{-2})^0}{(2x^4y)^3}$

Step 1: Apply the Zero-Exponent Rule. In this case, after applying the zero-exponent rule and multiplying by 1, that term is essentially gone.	$\frac{(-4x^3y^{-1})^2(1)}{(2x^4y)^3}$
Step 2: Apply the Power Rule. In this case, I kept the -2 in parentheses because I did not want to lose the negative sign.	$\frac{(-4)^2x^6y^{-2}}{2^3x^{12}y^3}$
Step 3: Apply the Negative Exponent Rule. Move every negative exponent from the numerator to the denominator and vice versa.	$\frac{(-4)^2x^6}{y^22^3x^{12}y^3}$
Step 4: Apply the Product Rule.	$\frac{(-4)^2x^6}{2^3x^{12}y^5}$
Step 5: Apply the Quotient Rule. In this case, the x's ended up in the denominator.	$\frac{(-4)^2}{2^3x^6y^5}$
Step 6: Raise each coefficient (or number) to the appropriate power and then simplify or reduce any remaining fractions. In this case, the fraction does reduce.	$\frac{16}{8x^6y^5} = \frac{2}{x^6y^5}$



LEVEL 5

WORK AREA 3: SHAPES & SPACE

Workshop Title: “Moving the shape without a change”
Duration: 5-7 hours
CODE: L5.WA3.N2
LEVEL 5
WORK AREA 3: SHAPES & SPACE
PARTNER: DIMITRA
Learning Outcomes Covered:
WORK AREA 3: SHAPES & SPACE LO1: Define the concepts of translation, reflection, rotation and dilation LO2: Present a geometric shape in a plane by using transformations.
Short introduction to the workshop
The workshop includes learning activities that refer to the different geometry shapes’ transformation. That will include learning activities focusing in reflection, rotation, translation and dilation. After the understanding of the concept of each transformation effect, a practical exercise will be given to the trainees for further understanding.
How the workshop (its activities) teach the LOs to the participants
Through this workshop, trainees will have a better understanding of the basic concepts of transformation and conceive their effect in a more substantial way in their everyday life (reflections, rotations etc.)
Challenges during the implementation of activities and how can be addressed
Participants, may confront many difficulties as the stage 5 is more challenging than former stages. Some of them can be either the difficulty in conceiving the different transformation effects or just find it challenging to distinguish among them.



ACTIVITY 1: “Dr Math introduces Transformations”

STEP 1: Define the concepts of translation, reflection, rotation and dilation through analytical PPT presentation.

STEP 2: Create four working stations a) reflection, b) rotation, c) translation, d) dilation, one at each corner of the training room and have the trainees go around and complete small activities concerning the concepts and their everyday use as described below.

Resources for Trainer

- Winpossible - Translation, Reflection, Dilation, and Rotation.
<https://www.youtube.com/watch?v=u6s4tEOEWJs> (accessed on 20/9/2019)

Other Resources needed

- Computer with possibility to play video



ACTIVITY 2: “Move me all over the place”

STEP 1: Give them a symmetric shape and ask them where they need to fold the paper in order for the shape to reflect itself. In continuance give them an asymmetric shape and ask them to do the same and discuss the difference.

STEP 2: Ask them to find examples of translation in everyday life and discuss them in the classroom.

Resources for Trainer

-

Other Resources needed

- Different pieces of symmetric papers
- Computer with possibility to play video



ACTIVITY 3: “You spin me right round”

STEP 1: Start with a Cartesian paper and a symmetric shape and ask them to rotate it 90 degrees, and then ask them to do the same with a non symmetrical object.

STEP 2: Ask them to find examples of rotation in everyday life and discuss them in the classroom.

Resources for Trainer

-

Other Resources needed

- Cartesian paper, paper cut in symmetric form
- Computer with possibility to play video



ACTIVITY 4: “The invisible mirror”

STEP 1: Ask the trainees to identify the concept of reflection by moving appropriately a polygon on a coordinate plane.

STEP 2: Ask them to find examples of reflection in everyday life and discuss them in the classroom.

Resources for Trainer

-

Other Resources needed

- Paper shaped in a polygon form
- Computer with possibility to play video



ACTIVITY 5: “Find the effect”

STEP 1: Draw different shape effects on a board (reflection rotation, translation, dilation) and ask the trainees to match the correct effect with each figure.

Resources for Trainer

- Winpossible - Translation, Reflection, Dilation, and Rotation.
<https://www.youtube.com/watch?v=u6s4tEOEWJs>
(accessed on 20/9/2019)

Other Resources needed

- A training room
- A whiteboard and markers



WORK AREA 4: HANDLING
INFORMATION

LEVEL 5

Workshop Title: Of hunting and gathering...data - and how to prepare something tasty with your ingredients
CODE:L5.WA4.N3
LEVEL 5
WORK AREA 4: HANDLING INFORMATION
PARTNER: MET
Duration: 7 hours
Learning Outcomes Covered:
WORK AREA 4: HANDLING INFORMATION Unit 4.3 Data processing LO1: Have knowledge of the basic six stages in data processing: • Data collection • Storage of data • Sorting of data • Processing of data • Data analysis • Data presentation & conclusions LO2: Utilize the data processing cycle • 3. Organize the collection of data • 4. Choose which method to use to process data • 5. Sort data out to start processing them • 6. Process data • 7. Draw conclusions after having processed selected data LO3: Present data using graphs, tables or charts
Short introduction to the workshop
This workshop is showing to the learners how to deal with given data in a structured way. They will get to know how to collect, store and sort it. Also they will learn how to analyse the data and illustrate the information gained. Learners will be enabled to know what methods of data collection and analyses are necessary. They will practice drawing conclusions and will be capable to explain and illustrate the results and conclusions in appropriate ways.



How the workshop (its activities) teach the LOs to the participants

There are two activities in this workshop. The first activity is intended to introduce the topic data processing. There is an introduction by means of a short video. Afterwards the learners are challenged to learn the single steps of data processing. Therefore they have to bring the steps in the right order by means of a worksheet.

In the second activity the learners have to apply this knowledge. Now they have to carry out their own data processing project and go through all the steps. They can choose the topic of their project from their everyday life. In this way the topic should be more realistic and of greater interest to them.

Challenges during the implementation of activities and how can be addressed

The second activity in particular requires much independent work from the learners. The teacher has to take care to keep an eye on all learners and to be a contact person if there are any questions. He also has to notice if a student and a group is not sure what to do or is doing the wrong thing. Even if the teacher himself is not pretending, a high participation is required. It is therefore especially important that he gives very precise and easy to understand instructions for the tasks and explains each step individually, one after the other and repeats the tasks.

Furthermore, it is very important for the success of this workshop that all learners have a PC and Internet access.



Step by step instructions for the trainers/mentors

ACTIVITY 1: Data, data and even more data and how to process them

Step 1: Watch a video introducing what data and information are and the data processing cycle (see Video 1 in Resources).

Step 2: The students are then asked to put the 6 steps of data processing in the right order. For this purpose, they should use the worksheet in Appendix 1.

Step 3: The students compare the answer with the teacher and discuss outstanding questions.

Resources for Trainer

- Video 1: <https://www.youtube.com/watch?v=8xoOLerFOwg>
- Worksheet from Annex 2 printed out for each learner or sent digitally to the learners for editing on the PC

Other Resources needed

- Possibility to play video and sound



ACTIVITY 2: Using data for a better understanding and presentation of facts

Step 1: Each learner is asked to choose a topic that he/she really likes and that they would like to know more about in terms of numbers (e.g. a hobby like football, dance etc., or about food, about children, about nature related topics, a country...). They can do it alone or in pairs. Enter the topic in the worksheet [Annex 1](#), as well as the following items.

Step 2: The trainer provides a few general guiding questions like: “How many....are there?”, “How often....?” etc. The learners need to specify more of these questions related to their topics. The trainer checks if the topic and the created questions is manageable in a given timeframe.

Step 3: The goal is to collect information, document the process and results and present the project to the others in the end. The students should therefore first collect data.

The trainer can help to select methods of collecting data appropriate for the topic. This can be a research in the internet; it can be an interview with someone, or watching a video, reading a newspaper, or an observation.

Step 4: In addition, the learners need to apply ways of storing the data, e.g. writing down, saving as a file, recording on a video, recording on audio etc. Again the trainer can help to decide.

Step 5: Next, Learners should sort the data and enter the results in the table (see [Annex 1](#)).

The learners gather the information in the way chosen. They will follow the cycle of data processing and apply for instance simple tables to organize their findings (categories / numbers). They will translate (describe) the numbers/data into words in order to describe them. They think of appropriate ways to illustrate the data (information), e.g. in a line graph showing the growth or increase during the past 5 years etc.

The organisation, processing and presentation of the data should primarily be done with the help of a computer. Writing, calculating and presentation programs of choice should be used.

Step 6: At the end the learners should present the results of their data processing project to others in the group. In particular, they should show the graphics, tables and diagrams they have created and derive the results from them.

Resources for Trainer

- Annex 2

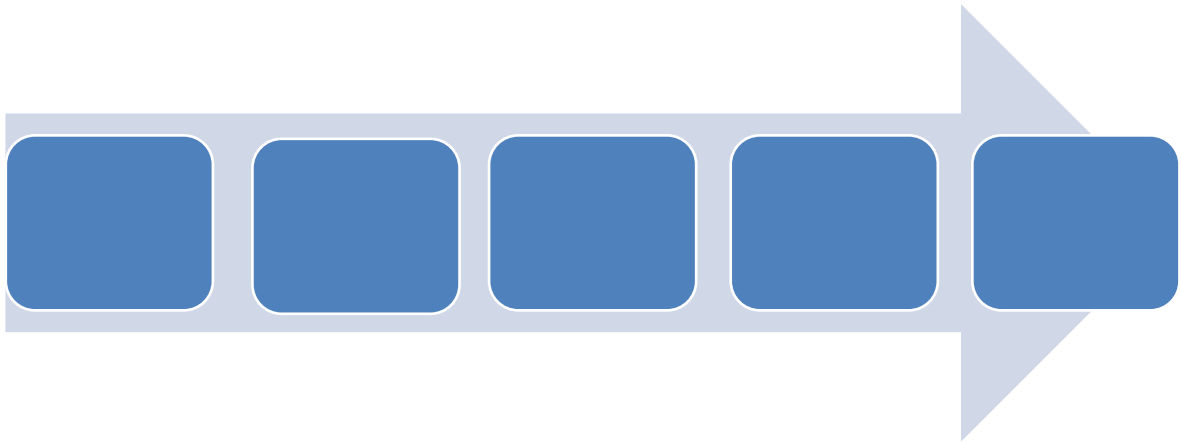
Other Resources needed

- Possibility for all learners to have access to computers and the Internet.



ANNEXES

Annex 1: 6 steps of Data processing



Enter the names of the individual data processing steps in the graphic in the correct order:

- **Data analysis:** In general, data analysis is about extracting information or insights from processed data. The data must be structured, ordered and presented using various techniques, such as statistical methods, so that the results serve as a basis for problem solving. The analysis can be qualitative or quantitative. Statistical data analysis, whether descriptive or explorative, is the basis for many areas of daily life, such as opinion polls.
- **Data collection:** the logically related data is collected from the different sources, different format, different types like from XML, CSV file, social media, images that is what structured or unstructured data and so all.
- **Processing of data:** A set of processing operations or continuous use and processing performed to verify, transform, organise, integrate and extract data in a useful output form for further use. This means preparing the data in such a way that it can continue to be used in a useful way.
- **Data presentation & conclusions:** Once we come to the analysis result, it can be displayed in various forms such as diagram, text file, Excel file, graphic and so on. The presentation methods must be determined according to the data format, the analysis method to be used and the information to be highlighted. With a single software or a combination of software, the saving, sorting, filtering and processing of data can be done, whatever is possible and necessary.
- **Sorting of data:** Sorting and filtering of the data is necessary to arrange the data in a meaningful order and to filter out only the necessary information that will later enable an easily understandable visualization and analysis.



- **Storage of data:** The data must be stored in physical forms such as paper, notebook and in all or any other physical form. Due to the often large amounts of data, the collection of data is very extensive, even in structured or unstructured form. The data should therefore be stored in digital form. This also facilitates later evaluation.



Annex 2: Data processing

My Topic (e.g. Football teams in the champions league):

Consider 3 questions on your topic (e.g. How many teams currently play in the Champions league? How often has the current best team won this season?):

1

2

3

Data collection – Search for data on your topic (List here where you can find data on your topic: Social media, newspapers, television):

Data storage (collect all your results in one place, e.g. cut out all relevant newspaper articles and collect them in a folder or copy links from websites into a Word document, save photos in a digital folder or copy text into a Word document)

Sorting of Data (select the photos or articles that are important in connection with your questions, or mark passages of text in colour to provide answers to your questions)

Processing of data (Put the data into a form that you can use to analyze it in the next step.)

Data analysis (Choose here how you want to analyze the data. Also use methods that you have already learned in other workshops. For example, you can do calculations like the mean or median.) You can enter your selected methods here:

Data presentation & conclusions (Use methods here again that you already know. For example, you can display the results in a pie chart or a bar chart.) You can enter your selected methods here:



WORK AREA 5: REASONING AND
PROBLEM SOLVING

LEVEL 5

Workshop Title: Brain Drain

CODE:L5.WA5.N1

LEVEL 5

WORK AREA 5: REASONING AND PROBLEM SOLVING

PARTNER: EDITC

Duration: 7 hours

Learning Outcomes Covered:

WORK AREA 5: REASONING & PROBLEM SOLVING

Unit 5.1 Problem Solving

- LO1. Provide a definition to a problem
- LO2. Describe goals and barriers in problem solving
- LO3. Identify goals and barriers in problem solving
- LO4. List the steps in problem solving
- LO5. Describe the steps in problem solving
- LO6. Apply the steps in problem solving
- LO7. Have knowledge of the methods to Structure the Problem
- LO8. Enlist possible visual ways to present a problem (i.e. Chain diagrams, Flow charts, Tree diagrams)
- LO9. Structure the Problem and Use the possible visual ways to present a problem (i.e. Chain diagrams, Flow charts, Tree diagrams)
- LO10. Outline methods to search for possible solutions such as brainstorming, divergent and convergent thinking, questioning assumptions
- LO11. Apply these methods(brainstorming, divergent and convergent thinking, questioning assumptions) to generate possible solutions
- LO12. Make rational decision based on risk assessment and evaluation of pros and cons
- LO13. Take responsibility
- LO14. Apply the steps for Implementing the decisions
- LO15. Monitoring/Seeking Feedback for improving the problem-solving techniques

Unit 5.2 Critical Thinking and reasoning

- LO1. Identify analogies
- LO2. Analyze analogies
- LO3. Create categories and classify items appropriately
- LO4. Identify relevant information
- LO5. Recognize valid deductive arguments, test hypotheses and recognize common reasoning fallacies.



- LO6. Construct valid deductive arguments, test hypotheses and recognize common reasoning fallacies
- LO7. Distinguish between evidence and interpretations of evidence

Short introduction to the workshop

How the workshop (its activities) teach the LOs to the participants

This workshop introduces a simple 4 steps method to problem solving. The first activity, **The strategy of solving Problems** starts with a very simple problem in order to go through the 4 step approach in solving problems. The second activity **“I need 38 grams”** is a more difficult one that aims to teach students on setting up a strategy for solving problems and providing them with the steps to implementing their strategy and finding a solution.

Challenges during the implementation of activities and how can be addressed



ACTIVITY 1: The strategy of solving Problems

Step 1: Introduce the 4 steps in solving Problems (see Annex 1). Explain the steps.

Step 2: Provide participants with the following problem

The Problem

I own 5 cars and a very large garage.

If I can see 2 cars parked outside the garage, how many are inside?

How many different ways can I park my cars inside and outside the garage?

Step 3: Read the first part of the problem to the class to ensure that they understand that they are working with 5 cars.

Step 4: Brainstorm for ways to solve the problem.

- Having students tell how they know the number of cars in the garage is the most important part of this problem. Allow the students to describe their ideas. Encourage explanations.
 1. *How did you know how many cars were hidden?*
 2. *Tell us about your thinking?*
 3. *Could there be any other number of cars in the garage when 2 are parked outside? How do you know?*

Step 5: Get the students to plan ways to record their solution.

Step 6: Read the second part of the problem and have students solve this in pairs or on their own. You need to use identical cars or there are multiple solutions for each pairing (for example: there would be 5 ways to complete the 1-4 pairing if all the cars were different). Support the students as they problem solve with questions such as:

- *How do you know how many cars are parked inside?*
Does there always have to be a car in the garage? or parked outside?
How do you know that you have found all the ways that the cars can be parked?
How are you keeping track of the ways that you find?
- Share and discuss solutions.



Step 6: The students compare the answer with the teacher and discuss outstanding questions.

Solution

Because $2 + 3 = 5$, if there are 2 cars inside the garage there must be 3 outside.
6 possibilities: (0,5) (1,4) (2,3) (3,2) (4,1) (5, 0)

Resources for Trainer

- Annex 1

Other Resources needed

- Whiteboard



ACTIVITY 2: I need 38 grams

Step 1: Ask the students to find an object that they estimate weighs 20g. Check estimates on the (balance) scales.

Step 2: Discuss students' ideas about how they made their estimates of 20g (eg, weight of small chip packet = 18g, flake bar = 30g). *What object in your desk would weigh close to 38g? How did you decide that? How do you use weights on a balance scale? How do you use these kitchen scales?*

Step 3: Pose the problem:

Dr Martin the chemist is weighing out some pills. He has some 5g weights and some 7g weights. Can he weigh exactly 38g of pills?

Step 4: As the students work on the problem in pairs, ask questions that focus their understanding of the size of grams. Focus their thinking on working systematically by asking questions about the way that they are keeping track of their work.

- *What are you doing?*
- *How will you share what you have done with others in the class.*
- *How do you know that you are on the right track?*

Step 5: Share solutions

Solution

38 is not exactly divisible by 5 or 7. Hence both 5g and 7g weights are needed. $38 - 7 = 31$, $38 - 2 \times 7 = 24$, and $38 - 3 \times 7 = 17$ are not divisible by 5. However, $38 - 4 \times 7 = 10 = 2 \times 5$. So Dr Martin can use four 7g weights and two 5g weights.

Step 6: Repeat the steps with the problems in **Annex 2**

Resources for Trainer

None

Other Resources needed

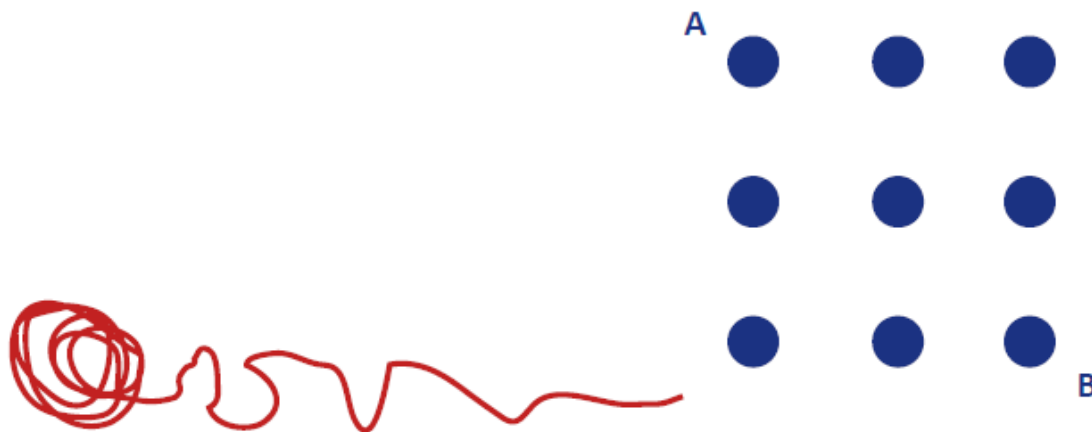
- Classroom
- Whiteboard and markers



ACTIVITY 3: Reasoning

Step 1: Present the following problem

Alex has a pegboard with 9 pegs in a 3 by 3 square array (see the diagram).



He also has a piece of string that he wants to put from the top left hand peg A, to the bottom right hand peg B, so that it touches all of the other pegs on the way only once.

If the string is never put diagonally between the pegs, how many different ways can Brian string up his pegboard?

Note: This problem challenges students in two ways. Firstly, they are challenged to be systematic in a novel situation. This is an important aspect of much of mathematics. Many students record their working in a haphazard way as they go along. This may lead to errors when students lose track of where they are and what they have done. Being systematic, and being careful with the recording of work, is an important tool that is fundamental to all mathematics. Secondly, this problem highlights the need to count in a situation that is new to them. Counting is a fundamental part of probability. In order to determine theoretical probabilities accurately, a knowledge of counting techniques is extremely important.

In this problem a guess and check strategy can be used. However, such an approach cannot guarantee that all outcomes have been obtained. Guess and check is nonetheless a good first strategy that will give the students an idea of the problem, along with some possible properties of the problem that they can use in a more systematic approach. However, students should be supported and encouraged to see that there is a better way to try to solve the problem. One way to do this is to help them to see where there are choices, and then to see what implications these choices have. (See solution.)

Once the students have the idea of being systematic in this problem, you might like encourage them further into an investigation. This might be done in several ways. One way is to look at any pair of pegs and see if they can be joined by string in a way prescribed.



Step 2: Let the students work in their groups to solve the problem. You may need to help them to see how to be systematic. If most of the class can only use guess and check, then it is worth gathering them all together to discuss how they might be systematic.

Step 3: Check the progress of each group. Give assistance where needed.

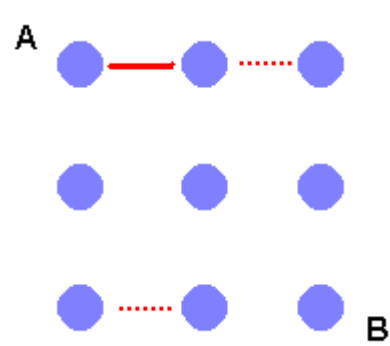
Step 4: Get the students to report back to the whole class.

Step 5: Give them time to write up their method of solution.

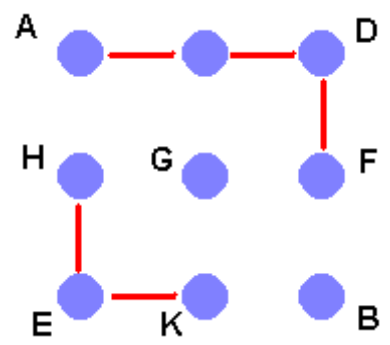
Step 6: Present step by step the solution

Solution

To be able to do this and make sure that we don't miss anything it is necessary to be systematic in what we do. To this end, there are a couple of things to notice straight away. First that we can start off either horizontally or vertically from A. But because of the symmetry of the square, whatever stringing we can do starting horizontally, there is another stringing that starts vertically. So we will explore the horizontal starts and then just rotate these at the end to give some more stringings. Let's assume then that we start as in the diagram.

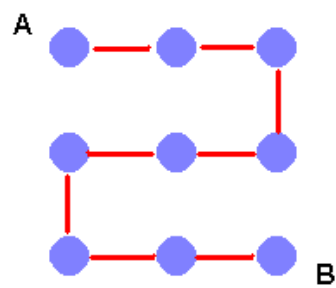


The second thing to notice is the limitations of the pegs D and E. There is only one way that the string can go round these pegs. We show this in the diagram below.



Now F can't join up to B as we have still more pegs to go past. So F joins to G.

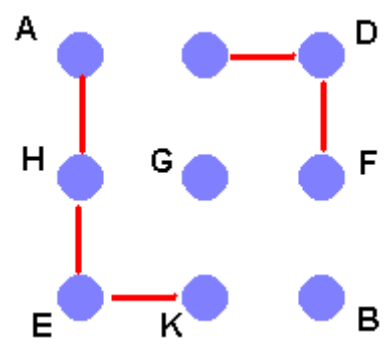
Then G joins H or else H is left high and dry. The only thing left is to join K and B to complete the stringing below.



Because of the rotation we talked about earlier, there is one other stringing. It is the rotation through 45° about the diagonal from A to B. This means that there are precisely two stringings that Alex can make altogether.

Solution to the extension:

Now what can be done here? First Alex can't join A to C as this misses out most of the pegs. So A joins H. And we still have the situation at D and E forced. So so far we have the situation below.



But the same thing has to happen at B as happens at D and E. This means that we can't use the peg. We can't join A to C in the way that we wanted.

What other pegs can A be joined to? What pairs of pegs **can** be joined by Alex?

Resources for Trainer

None

Other Resources needed

- Classroom
- Whiteboard and markers
- A peg and rope



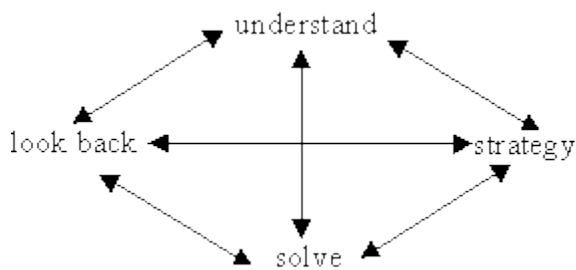
ANNEXES

ANNEX 1

Four Stages of Problem Solving

1. Understand and explore the problem;
2. Find a strategy;
3. Use the strategy to solve the problem
4. Look back and reflect on the solution.

Although we have listed the Four Stages of Problem Solving in order, for difficult problems it may not be possible to simply move through them consecutively to produce an answer. It is frequently the case that one may move backwards and forwards between and across the steps. In fact the diagram below is much more like what happens in practice



ANNEX 2

Problem 1

Mia and Hunter call a cab. The taxi driver charges \$1 flagfall when they get in the car. The charge is then \$2 for each kilometre they travel. They have \$23 between them. How far can they travel?

Solution

A table is an organized way to record the solution.

\$	1	3	5	7	9	11	13	15	17	19	21	23
km	0	1	2	3	4	5	6	7	8	9	10	11

So the friends can travel for 11 km in the taxi.

Problem 2

Maria is selling hats for the golf club. Below you see the number of hats that Maria sold during the first three weeks.

Week 1	         
Week 2	     
Week 3	  
Week 4	???

How many hats must Maria sell in week 4 so that the average number of caps that she has sold per week is 7?

Solution

The average is the number of caps sold altogether divided by the number of weeks (4). The given average is 7. The equation is $? \div 4 = 7$

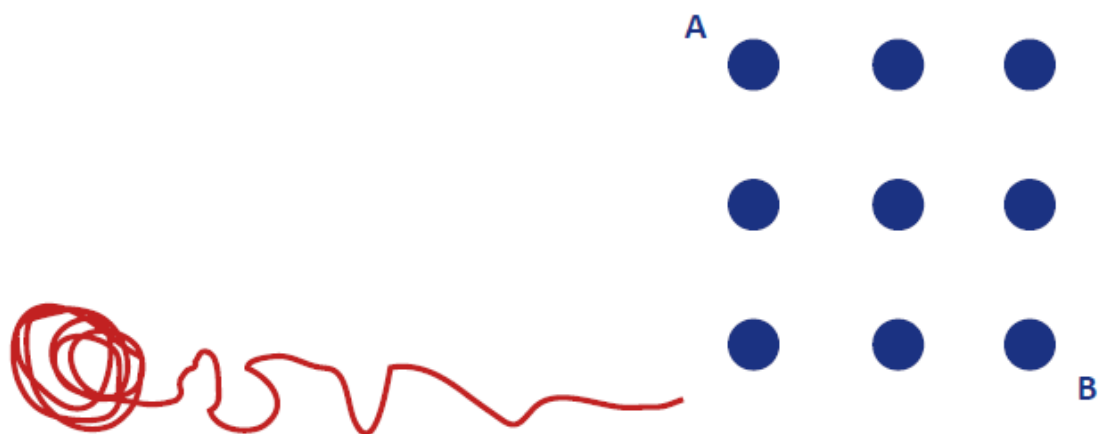
The inverse operation shows $4 \times 7 = 28$, so 28 caps are sold over the 4 weeks giving an average of 7 per week.



Maria sells 9 in the first week, 6 in the second and 3 in the third. This is a total of 18. She must sell 28 to keep the average of 7. So Maria must sell 10 caps in the last week.



Annex 3



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into d!gits
